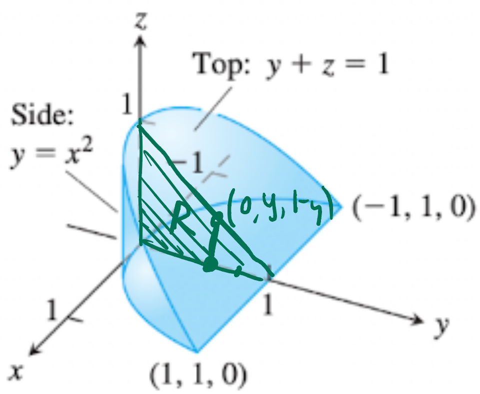
Quiz 5

Be sure to follow the quiz instructions in order to avoid a deduction in points. Submissions are due in Gradescope by 11:59pm on Friday; no late work is accepted.

Name: Key

Question #1: The triple integral evaluates to the volume of the region D pictured below. First (a) rewrite the triple integral in the order $dx\,dz\,dy$, and then (b) evaluate the expression from part (a).

$$\text{Vol} = \int_{-1}^{1}\int_{x^2}^{1}\int_{0}^{1-y} 1\,dz\,dy\,dx$$



(a) Vol= $\boxed{\int_0^1\int_0^{1-y}\int_{-\sqrt{y}}^{\sqrt{y}} 1\,dx\,dz\,dy}$

(b) Vol= $\boxed{8/15}$

$R\begin{cases} y \in [0,1] \\ z \in [0, 1-y] \end{cases}$

and $x \in [-\sqrt{y}, \sqrt{y}]$

So $\text{Vol} = \int_0^1\int_0^{1-y}\int_{-\sqrt{y}}^{\sqrt{y}} 1\,dx\,dz\,dy$

$= \int_0^1\int_0^{1-y} x\Big|_{-\sqrt{y}}^{\sqrt{y}}\,dz\,dy = \int_0^1\int_0^{1-y} 2\sqrt{y}\,dz\,dy$

$= \int_0^1 2\sqrt{y}\,z\Big|_0^{1-y}\,dz = \int_0^1 2\sqrt{y}(1-y)\,dy$

$= \int_0^1 2y^{1/2} - 2y^{3/2}\,dy = 2 * \frac{2}{3}y^{3/2} - 2 * \frac{2}{5}y^{5/2}\Big|_0^1$

$= \frac{4}{3} - \frac{4}{5} = \frac{20-12}{15} = \boxed{8/15}$

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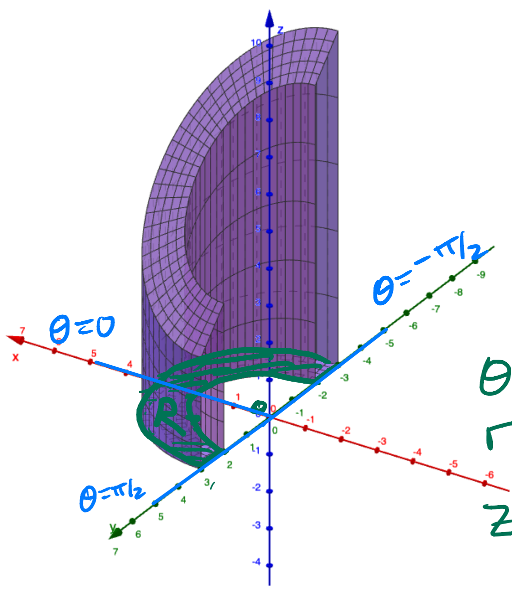
Question #2: Find the mass of the solid D pictured below, which occupies the region with base R the annulus $4 \le x^2 + y^2 \le 9$ **with** $x \ge 0$ in the xy-plane and with top cap the plane $y + z = 6$. First (a) write a triple integral in cylindrical coordinates which computes the mass of D given the density function $\delta(x, y, z) = 4\sqrt{x^2 + y^2}$ in the order $dz\,dr\,d\theta$, and then (b) evaluate the expression from part (a).



(a) mass= $\int_{-\pi/2}^{\pi/2}\int_{2}^{3}\int_{0}^{6 - r\sin\theta} 4r * r\,dz\,dr\,d\theta$

(b) mass= 152π

$\theta \in [-\frac{\pi}{2}, \frac{\pi}{2}]$

$r \in [2, 3]$

$z \in [0, 6 - y]$ w/ $y = r\sin\theta$

so $z \in [0, 6 - r\sin\theta]$.

$\delta = 4\sqrt{x^2+y^2} = 4r$ & $dA = r\,dz\,dr\,d\theta$

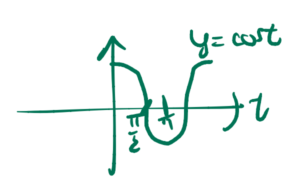
$$\text{mass} = \int_{-\pi/2}^{\pi/2}\int_{2}^{3}\int_{0}^{6 - r\sin\theta} 4r * r\,dz\,dr\,d\theta = \int\int_{2}^{3} 4r^2 z \Big|_{0}^{6 - r\sin\theta} dr\,d\theta$$

$$= \int_{-\pi/2}^{\pi/2}\int_{2}^{3} 4r^2(6 - r\sin\theta)\,dr\,d\theta = \int_{-\pi/2}^{\pi/2} \frac{24}{3}r^3 - \frac{4}{4}r^4\sin\theta \Big|_{2}^{3} d\theta$$

$$= \int_{-\pi/2}^{\pi/2} 8(3^3 - 2^3) - (3^4 - 2^4)\sin\theta\,d\theta = \int_{-\pi/2}^{\pi/2} 152 - 65\sin\theta\,d\theta$$

$$= 152\theta + 65\cos\theta \Big|_{-\pi/2}^{\pi/2} = 152\left(\frac{\pi}{2} - -\frac{\pi}{2}\right) + 65\left(\cancel{\cos\left(\frac{\pi}{2}\right)}^{0} - \cancel{\cos\left(-\frac{\pi}{2}\right)}^{0}\right)$$

$$= \boxed{152\pi}$$



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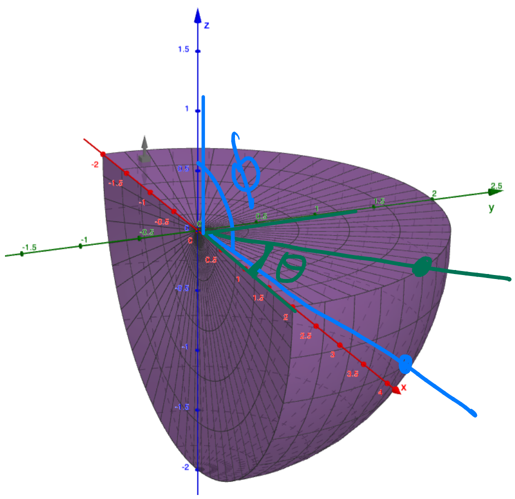
Question #3: Find the mass of the solid D pictured below, which occupies the region which is the right-half of the lower-half of the sphere of radius $\rho = 2$ with $y \geq 0$ and $z \leq 0$. First (a) write a triple integral in cylindrical coordinates which computes the mass of D given the density function $\delta(x, y, z) = -z$ in the order $d\phi\, d\rho\, d\theta$, and then (b) evaluate the expression from part (a).



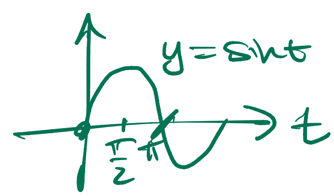
(a) mass= $\boxed{\int_0^{\pi}\int_0^{2}\int_{\pi/2}^{\pi} -\rho^3 \cos\phi \sin\phi \, d\phi\, d\rho\, d\theta}$

(b) mass= $\boxed{2\pi}$

$\theta \in [0, \pi]$ $\qquad \delta = -z = -\rho\cos\phi$

$\phi \in [\pi/2, \pi]$ $\qquad dA = \rho^2 \sin\phi \, d\phi\, d\rho\, d\theta$

$\rho \in [0, 2]$



So mass $= \int_0^{\pi}\int_0^{2}\int_{\pi/2}^{\pi} -\rho\cos\phi * \rho^2 \sin\phi \, d\phi\, d\rho\, d\theta$

$u = \sin\phi$
$du = \cos\phi\, d\phi$

$= \int_0^{\pi}\int_0^{2}\int_{\pi/2}^{\pi} -\rho^3 \sin\phi\cos\phi \, d\phi\, d\rho\, d\theta = \int_0^{\pi}\int_0^{\pi/2} -\frac{\rho^3}{2}\sin^2\phi \Big|_{\pi/2}^{\pi} d\rho\, d\theta$

$= \int_0^{\pi}\int_0^{2} -\frac{\rho^3}{2}\left(\overset{0}{\cancel{\sin^2\pi}} - \overset{1}{\cancel{\sin^2 \pi/2}}\right) d\rho\, d\theta = \int_0^{\pi}\int_0^{2} +\frac{\rho^3}{2}\, d\rho\, d\theta$

$= \int_0^{\pi} \frac{\rho^4}{8}\Big|_0^2 d\theta = \int_0^{\pi} 2\, d\theta = 2\theta\Big|_0^{\pi} = \boxed{2\pi}$