FORMULA SHEET

- Total Derivative: For $\mathbf{f}(x_1, \dots, x_n) = \langle f_1(x_1, \dots, x_n), \dots, f_m(x_1, \dots, x_n) \rangle$ $D\mathbf{f} = \begin{bmatrix} (f_1)_{x_1} & (f_1)_{x_2} & \dots & (f_1)_{x_n} \\ (f_2)_{x_1} & (f_2)_{x_2} & \dots & (f_2)_{x_n} \\ \vdots & \ddots & \dots & \vdots \\ (f_m)_{x_1} & (f_m)_{x_2} & \dots & (f_m)_{x_n} \end{bmatrix}$
- Linearization: Near \mathbf{a} , $f(\mathbf{x}) \approx L(\mathbf{x}) = f(\mathbf{a}) + Df(\mathbf{a})(\mathbf{x} \mathbf{a})$
- Chain Rule: If $h = g(f(\mathbf{x}))$ then $Dh(\mathbf{x}) = Dg(f(\mathbf{x}))Df(\mathbf{x})$
- Implicit Differentiation: If z is implicitly given in terms of x and y by F(x, y, z) = c, then $\frac{\partial z}{\partial x} = \frac{-F_x}{F_z}$ and $\frac{\partial z}{\partial y} = \frac{-F_y}{F_z}$.
- Directional Derivative: If **u** is a unit vector, $D_{\mathbf{u}}f(P) = Df(P)\mathbf{u} = \nabla f(P) \cdot \mathbf{u}$
- The tangent line to a level curve of f(x, y) at (a, b) is $0 = \nabla f(a, b) \cdot \langle x a, y b \rangle$
- The tangent plane to a level surface of f(x, y, z) at (a, b, c) is

$$0 = \nabla f(a, b, c) \cdot \langle x - a, y - b, z - c \rangle.$$

- Hessian Matrix: For f(x,y), $Hf(x,y) = \begin{bmatrix} f_{xx} & f_{yx} \\ f_{xy} & f_{yy} \end{bmatrix}$
- Second Derivative Test: If (a, b) is a critical point of f(x, y) then
 - 1. If det(Hf(a, b)) > 0 and $f_{xx}(a, b) < 0$ then f has a local maximum at (a, b)
 - 2. If det(Hf(a, b)) > 0 and $f_{xx}(a, b) > 0$ then f has a local minimum at (a, b)
 - 3. If det(Hf(a, b)) < 0 then f has a saddle point at (a, b)
 - 4. If det(Hf(a, b)) = 0 the test is inconclusive
- Area/volume: area $(R) = \iint_R dA$
- Trig identities: $\sin^2(x) = \frac{1}{2}(1 \cos(2x)), \quad \cos^2(x) = \frac{1}{2}(1 + \cos(2x))$
- Average value: $f_{avg} = \frac{\iint_R f(x, y) dA}{\text{area of } R}$
- Polar coordinates: $x = r \cos(\theta)$, $y = r \sin(\theta)$, $dA = r dr d\theta$