Math 2551 Worksheet Answers - Review from Single-Variable Calculus and Linear Algebra

- 1. $2\mathbf{v} = \langle 2, 4, 2 \rangle, \, \mathbf{v} \mathbf{w} = \langle 2, 2, -1 \rangle, \, \mathbf{v} \cdot \mathbf{w} = 1.$
- 2. a + 15b + 7c
- 3. $\mathbf{u} \cdot \mathbf{v} = 0$ (because the vectors are orthogonal)
- 4. (a) $\mathbf{v} \cdot \mathbf{u} = -25$ (b) $\cos(\theta) = \frac{-25}{25} = -1$ (c) $(3\mathbf{v}) \cdot (2\mathbf{u}) = 3(2)(-25) = -150.$
- 5. (a) 16/15
 - (b) $\frac{\pi}{2}$ (c) $\ln(4) - \frac{3}{4}$
- 6. $f_{max} = 2e^4$, since the function has no critical points and is increasing on this interval. 7. Area = $\frac{9}{2}$.

Math 2551 Worksheet Answers: \mathbb{R}^3

- 1. $x = 0, y = 3\sqrt{3}/2, z = 3/2.$
- 2. The first car is faster, at approx 43.28 km/h.
- 3. $(x-1)^2 + (y+2)^2 + (z-3)^2 = 16.$

Math 2551 Worksheet 2 Answers: \mathbb{R}^3 and Cross Products

- 1. Let P = (1, -1, 2), Q = (2, 0, -1), and R = (0, 2, 1).
 - (a) $2\sqrt{6}$ (b) $\frac{1}{\sqrt{6}}\langle 2, 1, 1 \rangle$
- 2. (a) True
 - (b) True
 - (c) True
- 3. (a) Makes sense, scalar
 - (b) Does not make sense
 - (c) Makes sense, vector
 - (d) Does not make sense

4. Max value is $\sqrt{13}$, given by $\mathbf{v} = \mathbf{u}/|\mathbf{u}|$ and min value is $-\sqrt{13}$, given by $\mathbf{v} = -\mathbf{u}/|\mathbf{u}|$.

Math 2551 Worksheet Answers: Lines and Planes

- 1. (a) $\mathbf{r}(t) = \langle 1 2t, 2 2t, -1 + 2t \rangle$ (b) $\mathbf{r}(t) = \langle t, -7 + 2t, 2t \rangle$
 - (c) $\mathbf{r}(t) = \langle 1 + 14t, 2t, 15t \rangle$
- 2. vector: $3\mathbf{i} 3\mathbf{j} + 3\mathbf{k}$. plane: 3(x-2) - 3(y-1) + 3(z+1) = 0
- 3. many correct solutions; pick two distinct points off of the line PQ which are not collinear with the given points. Two such planes are -2(x-1) + 3(y+1) + 5(z-1) = 0 and -3(x-1) + 3(y+1) + 3(z-1) = 0.
- 4. (2, 7, 3)
- 5. Parallel: $\langle A_1, B_1, C_1 \rangle = \lambda \langle A_2, B_2, C_2 \rangle$ for some $\lambda \neq 0$ Perpendicular: $\langle A_1, B_1, C_1 \rangle \cdot \langle A_2, B_2, C_2 \rangle = 0$
- 6. (4, 1, 3)

Math 2551 Worksheet Answers: Quadric Surfaces

- 1. (a) elliptical cylinder, oriented along the y-axis, cross-sections are ellipses in the y = k planes or vertical/horizontal lines in the x = k and z = k planes.
 - (b) ellipsoid, centered at the origin, wider in the z and y directions than the x direction, cross-sections are circles in the x = k planes (if k < 1), ellipses in the z = k and y = k planes (k < 3)
 - (c) circular cone, oriented along the y-axis, cross sections are circles in the y = k planes and lines in the x = k and z = k planes
 - (d) elliptical paraboloid, oriented in the positive z direction, shifted up 4 units, crosssections are circles in the z = k planes for $k \ge 4$ and parabolas in the x = k or y = kplanes
 - (e) sphere, centered at (0, 0, 0), radius 4, cross-sections are circles in x = k, y = k, and z = k planes for $0 \le k \le 4$.

Math 2551 Worksheet Answers: Curves in Space and Their Tangents

- 1. This curve's graph is a spiral, narrowing to a point at the origin when t = 0 and widening outward around the z-axis for larger/smaller t.
- 2. $\mathbf{r}(t) = \langle 3\cos(t), 3\sin(t), 2 3\sin(t) \rangle, 0 \le t \le 2\pi.$
- 3. All three functions describe part of the same set of points in \mathbb{R}^3 , which lie above the line y = x in the xy-plane and form a parabola in the plane x = y. **f** traces out all of the points on this parabola, **g** only those in the first octant, and **h** only those which lie above the square $[-1, 1] \times [-1, 1]$.
- 4. Many possible answers; depending on the domain and functions involved. If the domain is bounded, e.g. [a, b], then letting s = b + (a b)t and taking $\mathbf{r}(s)$ as the new parametric equations works. If the domain is $(-\infty, \infty)$, we can just let s = -t.
- 5. (a) $y = \frac{2}{9}x^2$ for x > 0(b) $\mathbf{v}(\ln(3)) = 3\mathbf{i} + 4\mathbf{j}$ $\mathbf{a}(\ln(3)) = 3\mathbf{i} + 8\mathbf{j}.$

6.
$$x(s) = s, y(s) = \frac{s}{3}, z(s) = s$$

7. (1,1,1) (where the first parameter is 1 and the second is 0). The angle is $\arccos(3/\sqrt{14})$. The bees would not collide, since the first bee reaches the point at t = 1 and the second bee at t = 0.

8.
$$-\frac{1}{\sqrt{2}}(x-\frac{1}{\sqrt{2}}) + \frac{1}{\sqrt{2}}(y-\frac{1}{\sqrt{2}}) + 6(z-0) = 0$$

OR $x - y - 6\sqrt{2}z = 0$

Math 2551 Worksheet 5 Answers: Calculus of Vector-Valued Functions

1.
$$\mathbf{r}(t) = \langle -\frac{1}{2}t^2 + 5t + 10, -\frac{1}{2}t^2 + 10, -\frac{1}{2}t^2 + 10 \rangle, \quad t \ge 0.$$

- 2. A baseball is hit when it is 2.5 ft above the ground. It leaves the bat with an initial velocity of 140 ft/sec at a launch angle of 30°. At the instant the ball is hit, an instantaneous gust of wind blows against the ball, adding a component of $-14\hat{i}$ (ft/sec) to the ball's initial velocity. A 15 ft high fence lies 400 ft from the home plate in the direction of the flight. (Note that gravity, g = 32 ft/sec²)
 - (a) Sorry, no sketch. :)
 - (b) $\vec{r}(t) = (140\cos 30^\circ 14)t\hat{i} + (2.5 + (140\sin 30^\circ)t 16t^2)\hat{j} = (70\sqrt{3} 14)t\hat{i} + (2.5 + (70t 16t^2)\hat{j})$
 - (c) $y_{\text{max}} = \frac{(140 \sin 30^{\circ})^2}{64} + 2.5 = \frac{70^2}{64} + 2.5 = 79.0625$ ft., which is reached at $t = \frac{140 \sin 30^{\circ}}{32} = \frac{70}{32} = 2.1875$ s.
 - (d) For the time, solve $y = 2.5 + 70t 16t^2 = 0$ for t. Using quadratic formula, we have t = 4.41s. Then, the range at t = 4.41 is $x(4.41) = (140 \cos 30^\circ 14)(4.41) = 472.94$ ft.
 - (e) For the time, solve $y = 2.5 + 70t 16t^2 = 20$ for t. Using quadratic formula, we have t = 0.27, 4.11 seconds. Then, the range at those times are x(0.27) = 29 ft and x(4.11) = 441 ft.
 - (f) Yes, according to part (d), the ball is still 20 feet above the ground when it is 441 feet from home plate.

Math 2551 Worksheet Answers: Arc Length

- 1. $\mathbf{T}(t) = \frac{1}{13} \langle 12\cos(2t), -12\sin(2t), 5 \rangle$ length: 13π
- 2. $(0, 5, 24\pi)$

3.
$$\int_0^1 \sqrt{(\frac{2}{t+1})^2 + (2e^{2t}+1)^2 + (2\sin(t)\cos(t))^2} dt$$

4. $\sqrt{6}$

Math 2551 Worksheet Answers: Curvature and Normals

1.
$$\mathbf{T}(t) = \frac{1}{1+e^{2t}} \langle \sqrt{2}e^t, e^{2t}, -1 \rangle$$
$$\mathbf{N}(t) = \frac{1}{1+e^{2t}} \langle 1-e^{2t}, \sqrt{2}e^t, \sqrt{2}e^t \rangle$$
$$\kappa(t) = \frac{\sqrt{2}e^{2t}}{(e^{2t}+1)^2}$$

2. $\mathbf{T}(t) = \cos(t)\mathbf{i} + \sin(t)\mathbf{j}$

$$\mathbf{N}(t) = -\sin(t)\mathbf{i} + \cos(t)\mathbf{j}$$
$$\kappa(t) = \frac{1}{t}$$

3. $\mathbf{T} = \langle \frac{1}{\sqrt{1+t^4}}, \frac{t^2}{\sqrt{1+t^4}} \rangle$ $\mathbf{N} = \langle \frac{-t^2}{\sqrt{1+t^4}}, \frac{1}{\sqrt{1+t^4}} \rangle \text{ if } t > 0 \text{ and } \langle \frac{t^2}{\sqrt{1+t^4}}, \frac{-1}{\sqrt{1+t^4}} \rangle \text{ if } t < 0.$

The normal vector does not exist when t = 0; as t passes from negative to positive values the normal vector changes which side of the curve it is on.

4. The point of greatest curvature occurs at x = 0. Using the parameterization $\mathbf{r}(t) = \langle t, t^2 \rangle$ gives $\kappa(t) = \frac{2}{(1+t^2)^{3/2}}$, which is maximized when t = 0.

Math 2551 Worksheet Answers: Multivariable Functions

- 1. Find and sketch the domain for each function.
 - (a) $\{(x, y) \mid x y \ge 1\}$
 - (b) $\{(x,y) \mid x \ge 4, |y| \ge 1\} \cup \{(x,y) \mid x < 4, |y| < 1\}$
 - (c) $\{(x,y) \mid 4x^2 1 \le y \le 4x^2 + 1\}$
 - (d) All of \mathbb{R}^2 except the circle $x^2 + y^2 = 4$
 - (e) All of the disk $x^2 + y^2 < 4$ except the circle $x^2 + y^2 = 3$.
- 2. (a) a collection of concentric ellipses
 - (b) a collection of unequally spaced concentric circles
 - (c) a collection of unequally spaced parallel lines
 - (d) a collection of equally spaced parallel lines
 - (e) a collection of unequally spaced concentric circles
 - (f) a collection of equally spaced concentric circles
 - (g) two straight lines and a collection of hyperbolas
- 3. The plane 4x + y + 3 = 0, except for those points with x + y = -1.
- 4. The sphere $3 = x^2 + y^2 + z^2$
- 5. When x = 0, the cross-section is the parabola $z = y^2$. When y = 0, the cross-section is the parabola $z = x^2$. When x = y, the cross-section is the line z = 0. The level curves are pairs of parallel lines $y = x \pm \sqrt{k}$.

Math 2551 Worksheet Answers: Limits and Continuity

- 1. $\frac{1}{36}$ 2. $\frac{1}{12}$ 3. $\frac{1}{4}$
- 4
- 4. Does not exist.
- 5. f is continuous on its entire domain: all (x, y) such that neither x = 0 nor y = 0.
- 6. f is continuous on its entire domain: all (x, y, z) except the sphere $x^2 + y^2 + z^2 = e$.

Math 2551 Worksheet Answers: Partial Derivatives

- 1. (a) $f_x(-2, -1) \approx 0.75$
 - (b) $f_y(-2, -1) \approx 1.5$
 - (c) There are several possible points (these are places where the tangent to a contour is horizontal): (0, -.5), (-.5, -1.25), etc.
 - (d) Again, there are many possible points; any point on the 4, 5, 6 contours in quadrant IV will work.
- 2. (a) C_T : (meters/second)/ degree Celsius this gives the change in speed for each one degree C of temperature increase. C_S (meters/second)/(grams/liter) - this gives the change in speed for each one gram/liter increase in salinity C_D : (meters/second)/meter - this gives the change in speed for each one meter increase in depth below the surface
 - (b) $C_T = 4.5 0.1T + 0.0009T^2 0.01(S 35) C_S = 1.5 0.01T C_D = 0.015$
 - (c) At (T, S, D) = (10, 35, 100), we have $C_T = 3.59, C_S = 1.4, C_D = 0.015$. This tells us that if we increase the temperature, salinity, or depth from these conditions the speed of sound will increase as well.

3.
$$f_{xx} = e^x, f_{xy} = f_{yx} = \frac{1}{y}, f_{yy} = -\frac{x}{y^2}$$

4.
$$f_x = \sin(yz), f_y = xz\cos(yz), f_z = xy\cos(yz), f_{xzz} = -y^2\sin(yz)$$

5. Find the total derivative Df at the given point for each function below. Remember that Df is the matrix of (partial) derivatives of the function and if f is a function from \mathbb{R}^n to \mathbb{R}^m then Df is a $m \times n$ matrix.

(a)
$$Df(2) = f'(2) = [24]$$

(b) $D\mathbf{f}(\pi/2) = \mathbf{f}'(\pi/2) = \begin{bmatrix} -2\\0\\1 \end{bmatrix}$
(c) $Df(1,2) = \begin{bmatrix} -1/2 & 1/2 \end{bmatrix}$
(d) $Df(1,2,3) = \begin{bmatrix} -e^3 & 2e^3 & 6 \end{bmatrix}$
(e) $D\mathbf{f}(1,1) = \begin{bmatrix} 2 & 3\\-1 & 1 \end{bmatrix}$

- 6. Note this does not have a definitive right answer some differences may arise and that's good! Discuss!
 - (a) First y since $\partial^3 f / \partial y^3 = 0$ and the y-partial derivatives are easier
 - (b) First y, since $\partial^3 f / \partial y^3 = 0$
 - (c) First y, since $\partial^2 f / \partial y^2 = 0$
 - (d) First x, since $\partial^2 f / \partial x^2 = 0$ and the x-partial derivatives are easier.

A common theme is to work with the variable with lower powers/simpler expressions first when taking mixed partials.

Math 2551 Worksheet Answers: Chain Rule

- $1. \ \frac{dz}{dt}(t_0) = 1$
- 2. t = -4/3
- 3. $\frac{\partial w}{\partial r}(2,\pi/2) = 2\pi$ $\frac{\partial w}{\partial \theta}(2,\pi/2) = -2\pi$
- 4. $\frac{dw}{dt}(2) = -10$. $\frac{dw}{dt}(1)$ cannot be computed from the given information because we do not know the values of g_x or g_y at (x(1), y(1)) = (1, 3). We do not use the values of $g(1, 0), g(-1, 2), g_x(-1, 2), g_y(-1, 2)$.

Math 2551 Worksheet Answers:Gradient and Directional Derivatives

- 1. (a) Negative
 - (b) Negative
 - (c) Approximately zero
 - (d) Positive
 - (e) Positive
- 2. Tangent line: -2(x-2) + 2(y+2) = 0



3.
$$D_{\mathbf{u}}g(1,-1) = \frac{21}{13}$$

- 4. (a) Ascend at a rate of 0.8 vertical meters per horizontal meter
 - (b) Descend at a rate of $\sqrt{2}/10$ vertical meters per horizontal meter
 - (c) $\langle -0.6, -0.8 \rangle$ is the direction of largest slope with rate of ascent 1 vertical meter per horizontal meter.
- 5. (a) $\langle -1/\sqrt{2}, 1/\sqrt{2} \rangle$
 - (b) $2\sqrt{2}$
 - (c) $\langle 1/\sqrt{2}, -1/\sqrt{2} \rangle$
 - (d) $\langle 1/\sqrt{2}, 1/\sqrt{2} \rangle$

Math 2551 Worksheet Answers: Linearization and Tangent Planes, Optimization

1. $L(x,y) = e^3 - e^3(x-1) + 2e^3(y-2)$

2.
$$L(x, y, z) = z$$

- 3. f(2.95, 7.1) = 4 1/40
- 4. No, there are two different tangent planes. No, the function is not differentiable.

5.
$$z = 1 - 1/2(x - 1) + 1/2(y - 2)$$

Math 2551 Worksheet Answers: Optimization I

- 1. Saddle point at (0,0) and local minimum at (0,-2).
- 2. Saddle point at (0,0) and local maximum at (-1,-1).
- 3. Yes, this must be a saddle point because $f_{xx}(a,b)f_{yy}(a,b) < 0$ so det $(Hf) = f_{xx}(a,b)f_{yy}(a,b) f_{xy}^2(a,b) < 0$.
- 4. (a) Minimum is 0 at (0,0) since f(x,y) > 0 for all other (x,y).
 - (b) Maximum is 1 at (0,0) since f(x,y) < 1 for all other (x,y).
 - (c) Neither since f(x, y) < 0 for x < 0 and f(x, y) > 0 for x > 0.
 - (d) Neither since f(x, y) < 0 for x < 0 and f(x, y) > 0 for x > 0.
 - (e) Neither since f(x, y) < 0 for x < 0 and y > 0, but f(x, y) > 0 for x > 0 and y > 0.
 - (f) Minimum is 0 at (0,0) since f(x,y) > 0 for all other (x,y).

Math 2551 Worksheet Answers: Optimization II

- 1. The absolute maximum is 17, achieved at (0, 4) and (4, 4), and the absolute minimum is 1, achieved at (0, 0).
- 2. The dimensions are $3 \times 3 \times 3$ and the surface area is 54.

Math 2551 Worksheet Answers: Lagrange Multipliers

- 1. V(2, 2, 1) = 4 cubic units
- 2. The extreme values are 1 and 2.
- 3. The extreme values are 1 and $\sqrt{3}$.
- 4. The closest point is $(\frac{1}{2}, \frac{1}{2}, \frac{1}{2})$ and the farthest point is (-1, -1, 2).
- $5. \ \frac{8r^3}{3\sqrt{3}}$

Math 2551 Worksheet Answers: Double Integrals on Rectangles



- 3. $9\ln(2)$
- 4. 160/3 cubic units.
- 5. $e^3 4$
- 6. The integrals evaluate to $\pi/4$ and $-\pi/4$ respectively. This does not violate Fubini's theorem because this function is not continuous on $[0, 1] \times [0, 1]$ (it has an asymptote at (0, 0))

Math 2551 Worksheet Answers: Double Integrals on General Regions

1. $2 + \frac{1}{2}\pi^2$

- 2. (a) Negative
 - (b) Positive
 - (c) Zero
 - (d) Zero
 - (e) Zero
- 3. (a) $\int_{0}^{\ln(3)} \int_{e^{-x}}^{1} dy dx$ and $\int_{1/3}^{1} \int_{-\ln(y)}^{\ln(3)} dx dy$ (b) $\int_{-1}^{2} \int_{x^{2}}^{x+2} dy dx$ and $\int_{0}^{1} \int_{-\sqrt{y}}^{\sqrt{y}} dx dy + \int_{1}^{4} \int_{y-2}^{\sqrt{y}} dx dy$ 4. $\int_{0}^{\sqrt{\pi}} \int_{0}^{x} \cos(x^{2}) dy dx = 0$ 5. $\frac{2}{15}$
- $6.\ 16/3$
- 7. 32/3

Math 2551 Worksheet 16 Answers: Applications, Polar Double Integrals

1. Answers will vary a bit through the estimation process

4 subdivisions: $31.75 \leq T_{avg} \leq 52.5$ 16 subdivisions: $33.18 \leq T_{avg} \leq 50.06$ 25 subdivisions: $36.32 \leq T_{avg} \leq 49.8$

Colorado is a rectangle, which makes it easy to subdivide. Wyoming would also work well.

2. On square: $f_{avg} = \frac{1}{1} \cdot \frac{1}{4} = \frac{1}{4}$

On quarter circle: $f_{avg} = \frac{1}{\pi/4} \cdot \frac{1}{8} = \frac{1}{2\pi}$

3. 50 people

Math 2551 Worksheet Answers: Polar Double Integrals

1.
$$5\pi - 26$$

2. $\frac{\pi}{2}(1 - e^{-4})$.
3. $\frac{3\pi}{2} - 4$.

4. 256π

5. (a)
$$I^2 = \lim_{R \to \infty} \int_0^2 \pi \int_0^R e^{-r^2} r \, dr \, d\theta$$

(b) $I^2 = \frac{\pi}{4}$, so $I = \frac{\sqrt{\pi}}{2}$

Math 2551 Worksheet Answers: Triple Integrals

- 1. $\frac{-58}{3}$, region of integration the rectangular prism $[-1, 1] \times [0, 4] \times [0, 1]$ or $-1 \le x \le 1, 0 \le y \le 4, 0 \le z \le 1$.
- 2. Various possibilities depending on order of integration. E.g.

$$\int_0^1 \int_0^{1-x} \int_0^{1-x-y} z \, dz \, dy \, dx$$

3. (a)
$$\int_{0}^{1} \int_{0}^{1} \int_{-1}^{-\sqrt{z}} dy \, dz \, dx$$

(b)
$$\int_{0}^{1} \int_{0}^{1} \int_{-1}^{1} \int_{-1}^{-\sqrt{z}} dy \, dx \, dz$$

(c)
$$\int_{0}^{1} \int_{-1}^{-\sqrt{z}} \int_{0}^{1} dx \, dy \, dz$$

(d)
$$\int_{-1}^{0} \int_{0}^{y^{2}} \int_{0}^{1} dx \, dz \, dy$$

(e)
$$\int_{-1}^{0} \int_{0}^{1} \int_{0}^{y^{2}} dz \, dx \, dy$$

4.
$$\int_{-1}^{1} \int_{1-\sqrt{1-x^{2}}}^{1+\sqrt{1-x^{2}}} \int_{x^{2}+y^{2}}^{2y} dz \, dy \, dx.$$

$$\int_{0}^{2} \int_{y^{2}}^{2y} \int_{-\sqrt{z-y^{2}}}^{\sqrt{z-y^{2}}} dx \, dz \, dy.$$

Yes, all orders of integration result in a single iterated integral.

Math 2551 Worksheet Answers: Mass and Moments

1.
$$(\bar{x}, \bar{y}) = \left(0, \frac{9}{14}\right)$$

2. Mass: $\int_{-2}^{2} \int_{-\sqrt{1-x^{2}/4}}^{\sqrt{1-x^{2}/4}} \int_{0}^{2-x} 2 \, dz \, dy \, dx$
 M_{yz} : $\int_{-2}^{2} \int_{-\sqrt{1-x^{2}/4}}^{\sqrt{1-x^{2}/4}} \int_{0}^{2-x} 2x \, dz \, dy \, dx$
 M_{xz} : $\int_{-2}^{2} \int_{-\sqrt{1-x^{2}/4}}^{\sqrt{1-x^{2}/4}} \int_{0}^{2-x} 2y \, dz \, dy \, dx$
 M_{xy} : $\int_{-2}^{2} \int_{-\sqrt{1-x^{2}/4}}^{\sqrt{1-x^{2}/4}} \int_{0}^{2-x} 2z \, dz \, dy \, dx$

Math 2551 Worksheet Answers: Triple Integrals in Cylindrical & Spherical Coordinates

1. Integral:
$$\int_{-\pi/2}^{\pi/2} \int_{0}^{1} \int_{0}^{r\cos(\theta)} r^{3} dz dr d\theta$$
Answer: $\frac{2}{5}$.
2. Integral:
$$\int_{0}^{\pi/6} \int_{0}^{\pi/2} \int_{0}^{a} \rho^{2} \sin(\phi) d\rho d\phi d\theta$$
Answer: $\frac{a^{3}\pi}{18}$.
3. Integral:
$$\int_{0}^{2\pi} \int_{\pi/4}^{\pi/2} \int_{0}^{2} \rho^{2} \sin(\phi) d\rho d\phi d\theta$$
Answer: $\frac{8\sqrt{2}\pi}{3}$
4. Integral:
$$\int_{0}^{2\pi} \int_{1}^{3} \int_{0}^{9-r^{2}} r dz dr d\theta$$
Answer: 32π
5. Integral:
$$2 \left(\int_{0}^{2\pi} \int_{\pi/6}^{\pi/4} \int_{\csc(\phi)}^{2} \rho^{2} \sin(\phi) d\rho d\phi d\theta + \int_{0}^{2\pi} \int_{\pi/4}^{\pi/2} \int_{\sqrt{2}}^{2} \rho^{2} \sin(\phi) d\rho d\phi d\theta \right)$$
Answer: $\frac{12\sqrt{3}-4}{3}\pi$.
6. (a) Integral:
$$\int_{0}^{\pi} \int_{0}^{2\sin(\theta)} \int_{0}^{4-r\sin(\theta)} r dz dr d\theta$$
Answer: 3π .
(b) $\bar{x} = 0$.

Math 2551 Worksheet Answers: Change of Variables

- 1. $\sin^2(\theta) \cos^2(\theta) = -\cos(2\theta)$
- 2. The elliptical region $\frac{x^2}{a^2} + \frac{y^2}{b^2} \le 1$.

Many possible answers: any such should involve an integration problem where the region of integration is an ellipse with major and minor axes parallel to the x and y-axes.

3.
$$x = \sqrt{\frac{u}{v}}, y = \sqrt{uv} \text{ maps } [1,3] \times [1,3] \text{ onto } R$$

4. $\int_{0}^{1} \int_{-3v}^{-3v+3} -2v \ du \ dv$
5. $\ln(9)$
6. $e + \frac{1}{e}$

Math 2551 Worksheet Answers: Scalar Line Integrals

- 1. There are many possible correct answers! Here are some.
 - (a) $\mathbf{r}(t) = \langle 3t, -2t+1, 4t-2 \rangle, \quad 0 \le t \le 1$
 - (b) $\mathbf{r}(t) = \langle -3t + 3, 2t 1, -4t + 2 \rangle, \quad 0 \le t \le 1$
 - (c) $\mathbf{r}(t) = \langle 3\sin(t), 3\cos(t) \rangle, \quad \pi \le t \le 3\pi$
 - (d) $\mathbf{r}(t) = \langle t^2, -t \rangle \quad -2 \le t \le 1$
- 2. The integral to compute is $\int_0^{2\pi} 20 \, dt$ since $f(\mathbf{r}(t)) = 4$ and $\|\mathbf{r}'(t)\| = 5$. Its value is 40π .
- 3. The integral to compute is $Ds \int_{-2}^{1} 4t^2 + 1 dt$. Its value is 15.
- 4. The integral to compute is $Ds \int_0^{\pi} e^{-t} e^t \sqrt{3} dt \sqrt{3}\pi$

Without rescaling vectors to fit better:

6.

With rescaling:



Math 2551 Worksheet Answers: Vector Line Integrals

- Top left: 0
 Bottom left: Negative Center: Positive Top right: 0
 Bottom right: Negative
- $2. \ 1$
- 3. $\frac{25}{6}$
- 4. Circulation: $\frac{32}{3}$ Flux: 0
- 5. Many possible examples: the top left and top right examples from 1), ∇f for any f together with a closed curve \mathbf{r} , any example where $\mathbf{F} \cdot \mathbf{r}'(t) = 0$, i.e. the field and curve are orthogonal, and more.

Math 2551 Worksheet Answers: Potentials and Conservative Vector Fields

1.
$$f(x,y) = 12x^2y + 2y^3$$

2.
$$f(x, y, z) = x^2 y - z^2 y$$
 and $\int_C \mathbf{F} \cdot d\mathbf{r} = -16$.

3. 7 = a, 2 = c, 4 = d, no restriction on b or e.

4.
$$f(x, y, z) = \frac{x}{y} + z^2 - z.$$
$$\int_C \mathbf{F} \cdot d\mathbf{r} = 1/2$$

5.
$$2\pi$$

Math 2551 Worksheet Answers: Curl, Divergence, Green's Theorem

- 1. $\nabla \cdot \mathbf{F}$ is 0 in all quadrants. $(\nabla \times \mathbf{F}) \cdot \mathbf{k}$ is negative in all quadrants.
- $\begin{array}{ll} 2. \ \, \nabla\cdot\mathbf{F}=0 \\ (\nabla\times\mathbf{F})\cdot\mathbf{k}=-\frac{1}{4}. \end{array}$
- 3. (a) $\mathbf{r}(t) = \langle 3\cos(t), 4\sin(t) \rangle, \ 0 \le t \le 2\pi.$
 - (b) Circulation: 0 Flux: 48π
- 4. Circulation: -44/15Flux: The integrand is very difficult to work with, so we should not use Green's theorem here.
- 5. -1/12
- 6. One answer (linear algebra!) $\mathbf{r}(s,t) = s\langle 1,-1,0 \rangle + t\langle 0,1,-1 \rangle = \langle s,t-s,-t \rangle, s,t \in \mathbb{R}.$

Math 2551 Worksheet Answers: Surfaces

- 1. One answer: $\mathbf{r}(u, v) = \langle u, 4 u^2, v \rangle, -2 \le u \le 2, 0 \le v \le 2.$
- 2. One answer: $\mathbf{r}(\phi, \theta) = \langle 2\sin(\phi)\cos(\theta), 2\sin(\phi)\sin(\theta), 2\cos(\phi) \rangle$ with $0 \le \phi \le \pi/6$ and $0 \le \theta \le 2\pi$. SA: $8\pi \left(1 - \frac{\sqrt{3}}{2}\right)$
- 3. The tangent plane is $-\sqrt{2}(x-\sqrt{2}) \sqrt{2}(y-\sqrt{2}) + 2(z-2) = 0$. This is the cone $z = \sqrt{x^2 + y^2}$.
- 4. $\frac{2}{3}\pi(2^{3/2}-1)$ 5. $\frac{\pi}{6}(5^{3/2}-1)$

Math 2551 Worksheet Answers: Surface Integrals

- 1. 0
- 2. $\frac{10\pi}{3}$
- 3. 0

Math 2551 Worksheet Answers: Stokes' Theorem

1. H and P have the same oriented boundary curve C provided that they are oriented in the same way, so by Stokes' theorem the given integrals must be equal.

2. 0

3. 3