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PA#4B- §14.8: Lagrange multipliers

Use the method of Lagrange multipliers to find the absolute maxima and absolute minima of the function $f(x, y) = x^2y$ subject to the constraint that $x^2 + y^2 = 1$.

$$\nabla f = \begin{bmatrix} 2xy \\ x^2 \end{bmatrix} \qquad \nabla g = \begin{bmatrix} 2x \\ 2y \end{bmatrix}$$

So Lagrange eqns are

$$\begin{cases} ① & 2xy = \lambda 2x \\ ② & x^2 = \lambda 2y \\ ③ & x^2 + y^2 = 1 \quad \text{(constraint)} \end{cases}$$

Case 1: $x=0$

If $x=0$ then $y = \pm 1$ from ③

So get two points $(0,1)$ & $(0,-1)$

Case 2: $x \neq 0$

If $x \neq 0$ then from ① get $2y = 2\lambda$ so $\lambda = y$

and sub into ② to get $x^2 = 2y^2$.

Now sub into ③ to get $(2y^2) + y^2 = 1 \Rightarrow 3y^2 = 1$

$\Rightarrow y = \pm\sqrt{1/3}$

So $x = \pm\sqrt{2/3}$ and get four points $(\pm\sqrt{2/3}, \sqrt{1/3})$ and $(\pm\sqrt{2/3}, -\sqrt{1/3})$

plug in all six points into $f(x,y) = x^2y$

(x,y)	$f(x,y) = x^2y$	
$(0,1)$	0	
$(0,-1)$	0	
$(\pm\sqrt{2/3}, \sqrt{1/3})$	$\frac{2}{3} * \sqrt{1/3}$	← MAX
$(\pm\sqrt{2/3}, -\sqrt{1/3})$	$\frac{2}{3} * -\sqrt{1/3}$	← MIN

Max value of $2/3\sqrt{3}$

and Min value of $-2/3\sqrt{3}$