

Taker Name:

GTID: 90

Section:

Grader #1:

GTID: 90

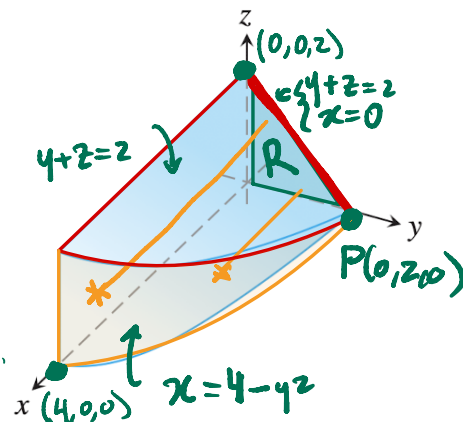
PA#5A - §15.5: Triple integrals in rectangular coordinates

Set up and evaluate a triple iterated integral in cartesian (rectangular) coordinates which computes the volume of the region  $D$ . The region  $D$  is the region bounded by the coordinate planes, the plane  $y+z=2$ , and the cylinder  $x=4-y^2$ . Hint:  $dV = dz dx dy$ .

P intersection point on plane and on  $y$ -axis, so  $x=0$  and  $z=0$ .

plane eqn  $y+z=2$  @  $z=0$  becomes  $y=2$ .

So  $P(0,2,0)$ .



So region  $R$  in  $yz$ -plane is  $y \in [0, z]$  and  $z \in [0, 2-y]$

Then to get solid region  $D$  extend in  $x$ -direction from  $R$  to get  $x \in [0, 4-y^2]$

$$\text{So } \text{Vol}(D) = \iiint_D 1 \, dV = \int_0^2 \int_0^{2-y} \int_0^{4-y^2} 1 \, dx \, dz \, dy$$

$$= \int_0^2 \int_0^{2-y} x \Big|_0^{4-y^2} \, dz \, dy = \int_0^2 \int_0^{2-y} 4-y^2 \, dz \, dy$$

$$= \int_0^2 4z - y^2 z \Big|_0^{2-y} \, dy = \int_0^2 4(2-y) - y^2(2-y) \, dy = 20/3$$

$$= \int_0^2 8 - 4y - 2y^2 + y^3 \, dy = 8y - 2y^2 - \frac{2}{3}y^3 + \frac{1}{4}y^4 \Big|_0^2 = 16 - 8 - \frac{16}{3} + 4 = \frac{36-16}{3}$$

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G2:

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G3:

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