

Taker Name:

Key

GTID: 90

Section:

Grader #1:

GTID: 90

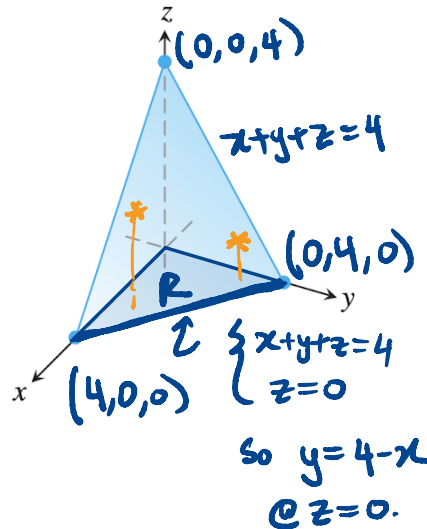
PA#5B - §15.5: Triple integrals in rectangular coordinates

Set up and evaluate a triple iterated integral in cartesian (rectangular) coordinates which computes the volume of the region D . The region D is the tetrahedron in the first octant bounded by the coordinate planes and the plane $x + y + z = 4$. Hint: $dV = dz dy dx$.

x -axis intersection @ $y=0$ and $z=0$

So $x=4$.

region R in xy -plane
defined by $x \in [0, 4]$ and
 $y \in [0, 4-x]$



For points in region R extend in the
 z -direction by $z \in [0, 4-x-y]$.

$$\text{So } \text{Vol}(D) = \iiint_D 1 \, dV = \int_0^4 \int_0^{4-x} \int_0^{4-x-y} 1 \, dz \, dy \, dx$$

$$= \int_0^4 \int_0^{4-x} z \Big|_0^{4-x-y} \, dy \, dx = \int_0^4 \int_0^{4-x} 4-x-y \, dy \, dx$$

$$= \int_0^4 4y - xy - \frac{1}{2}y^2 \Big|_0^{4-x} \, dx = \int_0^4 4(4-x) - x(4-x) - \frac{1}{2}(4-x)^2 \, dx$$

$$= \int_0^4 16 - 4x - 4x + x^2 - 8 + 4x - \frac{1}{2}x^2 \, dx = \int_0^4 8 - 4x + \frac{1}{2}x^2 \, dx$$

$$= 8x - 2x^2 + \frac{1}{6}x^3 \Big|_0^4 = 32 - 32 + \frac{1}{6}4^3 = 64/6 = \boxed{32/3}$$

A	
J	
N	
G2:	
A	
J	
N	
G3:	
A	
J	
N	