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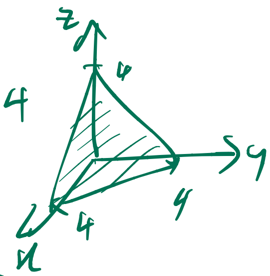
PA#7A - §16.6: Surface Integrals

Suppose the density of a thin plate S in the shape of the portion of the plane $z = 4 - x - y$ in the first octant is given by $\delta(x, y, z) = xy$. Find the mass of the plate.

$$\text{Mass} = \iint_S \delta \, d\sigma$$

$x + y + z = 4$



$r(s,t) = \langle s, t, 4-s-t \rangle$, $s \in [0,4]$, $t \in [0, 4-s]$

$r_s = \langle 1, 0, -1 \rangle$, $r_t = \langle 0, 1, -1 \rangle$

$$r_s \times r_t = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 0 & -1 \\ 0 & 1 & -1 \end{vmatrix} = \langle -(-1), -(-1), 1 \rangle = \langle 1, 1, 1 \rangle$$

$\|r_s \times r_t\| = \sqrt{3}$

$$\text{Mass} = \iint_S \delta \, d\sigma = \iint_R \delta * \sqrt{3} \, dA = \int_0^4 \int_0^{4-s} st * \sqrt{3} \, dt \, ds$$

$$= \int_0^4 \frac{\sqrt{3}}{2} t^2 s \Big|_0^{4-s} ds = \int_0^4 \frac{\sqrt{3}}{2} (4-s)^2 s \, ds$$

$$= \int_0^4 \frac{\sqrt{3}}{2} (s^2 - 8s + 16) s \, ds = \frac{\sqrt{3}}{2} \int_0^4 s^3 - 8s^2 + 16s \, ds$$

$$= \frac{\sqrt{3}}{2} \left(\frac{1}{4} s^4 - \frac{8}{3} s^3 + 8s^2 \right) \Big|_0^4 = \frac{\sqrt{3}}{2} \left(4^3 - \frac{8}{3} * 4^3 + 8 * 4^2 \right)$$

$$= \frac{\sqrt{3}}{2} 4^3 \left(1 - \frac{8}{3} + 2 \right) = \frac{\sqrt{3}}{2} * 4^3 * \frac{1}{3} = \boxed{\frac{32\sqrt{3}}{3}}$$