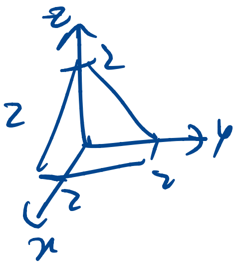
MATH 2551 / / **Peer Assessment Activity** SPRING '26

Taker Name:		GTID: 90	Section:
Grader #1:		GTID: 90	

PA#7B - §16.6: Surface Integrals

Suppose the density of a thin plate S in the shape of the portion of the plane $x + y + z = 2$ in the first octant is given by $\delta(x, y, z) = x + y$. Find the mass of the plate.

$x+y+z=2$



$$\text{Mass} = \iint_S \delta \, d\sigma$$

$r(s,t) = \langle s, t, 2-s-t \rangle$, $s \in [0,2]$, $t \in [0, 2-s]$

$r_s = \langle 1, 0, -1 \rangle$, $r_t = \langle 0, 1, -1 \rangle$

$$r_s \times r_t = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 0 & -1 \\ 0 & 1 & -1 \end{vmatrix} = \langle -(-1), -(-1), 1 \rangle = \langle 1, 1, 1 \rangle$$

$\|r_s \times r_t\| = \sqrt{3}$

$$\text{Mass} = \iint_S \delta \, d\sigma = \iint_R \delta \cdot \sqrt{3} \, dA = \int_0^2 \int_0^{2-s} (s+t)\sqrt{3} \, dt \, ds$$

$$= \int_0^2 \sqrt{3}\, st + \frac{\sqrt{3}}{2} t^2 \Big|_0^{2-s} ds$$

$$= \int_0^2 \sqrt{3}\, s(2-s) + \frac{\sqrt{3}}{2}(2-s)^2 \, ds = \int_0^2 2\sqrt{3}\, s - \sqrt{3}\, s^2 + \frac{\sqrt{3}}{2}(s^2 - 4s + 4) \, ds$$

$$= \int_0^2 \cancel{2\sqrt{3}\, s} - \sqrt{3}\, s^2 + \frac{\sqrt{3}}{2} s^2 - \cancel{2\sqrt{3}\, s} + 2\sqrt{3} \, ds = \int_0^4 -\frac{\sqrt{3}}{2} s^2 + 2\sqrt{3} \, ds$$

$$= -\frac{\sqrt{3}}{6} s^3 + 2\sqrt{3}\, s \Big|_0^2 = -\frac{\sqrt{3}}{6}(8) - 4\sqrt{3} = \sqrt{3}\left(4 - \frac{4}{3}\right) = \boxed{\frac{8\sqrt{3}}{3}}$$

A	
J	
N	

G2:

A	
J	
N	

G3:

A	
J	
N	