MATH 2551 J/HP Midterm 1
VERSION A
Spring 2026
COVERS SECTIONS 12.1-12.5, 13.1-13.4, 14.1-14.2

Full name: ______________________________ **GT ID:**____________

Honor code statement: I will abide strictly by the Georgia Tech honor code at all times. I will not use a calculator. **I do not have a phone within reach**, and I will not reference any website, application, or other CAS-enabled service. I will not consult with my notes or anyone during this exam. I will not provide aid to anyone else during this exam.

(   ) All of the knowledge presented in this exam is entirely my own. I am initialing to the left to attest to my integrity.

Read all instructions carefully before beginning.

- Print your name and GT ID neatly above.
- You have 75 minutes to take the exam.
- You may not use aids of any kind.
- Please show your work [J] and annotate your work using proper notation [N].
- Good luck!

Question	Points
1	2
2	2
3	4
4	8
5	10
6	8
7	8
8	8
Total:	50

For T/F problems choose whether the statement is true or false. If the statement is *always* true, pick true. If the statement is *ever* false, pick false. Also please be sure to neatly fill in the bubble corresponding to your answer choice. [A]

1. (2 points) If $\mathbf{u}$ and $\mathbf{v}$ are orthogonal unit vectors in $\mathbb{R}^3$, then $\|\mathbf{u} \times \mathbf{v}\| = 1$.

   ◯ **TRUE** ◯ **FALSE**

2. (2 points) Which of the following vectors could be the principal unit normal vector at time $t = 2$ to a curve with parametrization $\mathbf{r}(t)$ whose tangent line at $t = 2$ is given by

$$\ell(s) = \langle 1, 1, 1 \rangle + s\langle 2, 1, 2 \rangle, \; s \in \mathbb{R}.$$

   You do not need to justify your answers. *Select all that apply.* [A]

   ◯ **A)** $\langle 2, -1, 2 \rangle$
   ◯ **B)** $\left\langle \frac{2}{3}, -\frac{1}{3}, \frac{2}{3} \right\rangle$
   ◯ **C)** $\left\langle \frac{1}{3}, \frac{2}{3}, -\frac{2}{3} \right\rangle$
   ◯ **D)** $\left\langle \frac{1}{\sqrt{5}}, -\frac{2}{\sqrt{5}}, 0 \right\rangle$
   ◯ **E)** $\left\langle \frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}} \right\rangle$

3. (4 points) Find the area of the parallelogram determined by $P(1, 1, 1)$, $Q(2, 3, 1)$, $R(4, 2, 1)$, and $S(5, 4, 1)$. [AJN]

4. (8 points) Let P be the plane defined by $x - 2y + 3z = 25$. Find (a) the vector equation for the line ℓ passing through the point $Q(2, 0, 1)$ which is orthogonal to P, and (b) find the intersection between this line ℓ and the plane P.

[AJN]

(a)

(b)

5. (10 points) Let $\mathbf{r}(t) = \langle 2\cos 2t, 2\sin 2t, 4t\rangle$, $0 \le t \le \pi$. Find (a) the curve's unit tangent vector $\mathbf{T}(t)$ and (b) the length of the curve parametrized by $\mathbf{r}(t)$. [AJN]

$\mathbf{T}(t) =$

length is

6. (8 points) In this problem, you will work with the curve

$$\mathbf{r}(t) = t\mathbf{i} + (\ln\cos t)\mathbf{j}$$

for $-\pi/2 < t < \pi/2$. [AJN]

(a) Compute the principal unit normal vector $\mathbf{N}(t)$.

(b) Compute the curvature $\kappa(t)$.

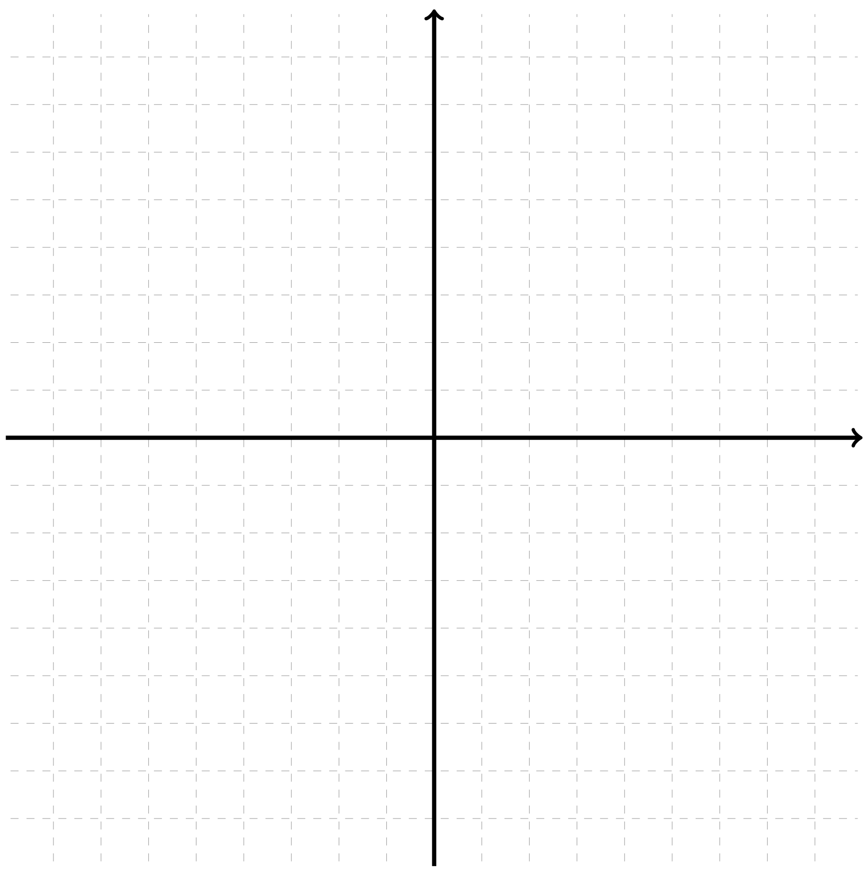
$\mathbf{N}(t) =$ [ ]

$\kappa(t) =$ [ ]

7. (8 points) Draw a contour map on the axes provided including all three of the level curves $g(x,y) = c$ for the function

$$g(x,y) = 4 - x^2 - y, \quad c = -5, 0, 3.$$

Show your work for how you find the equation of each level set, include labels for the axes, and label each level set as well as any x-intercepts or y-intercepts of each level set. [AJN]



8. (8 points) Show that the limit does not exist. To receive full credit, you must show work supporting your answer, use proper limit notation, and mention the test that you are using. [AJN]

$$\lim_{(x,y)\to(0,0)} \frac{x^2y}{x^4+xy^2}$$

FORMULA SHEET

- $\langle u_1, u_2, u_3 \rangle \cdot \langle v_1, v_2, v_3 \rangle = u_1 v_1 + u_2 v_2 + u_3 v_3$
- $\mathbf{u} \cdot \mathbf{v} = |\mathbf{u}||\mathbf{v}| \cos(\theta)$
- $\langle u_1, u_2, u_3 \rangle \times \langle v_1, v_2, v_3 \rangle = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ u_1 & u_2 & u_3 \\ v_1 & v_2 & v_3 \end{vmatrix}$
- $|\mathbf{u} \times \mathbf{v}| = |\mathbf{u}||\mathbf{v}||\sin(\theta)|$
- $L = \displaystyle\int_a^b |\mathbf{r}'(t)| \; dt$
- $s(t) = \displaystyle\int_{t_0}^t |\mathbf{r}'(\tau)| \; d\tau$
- $\mathbf{T} = \dfrac{\mathbf{v}}{|\mathbf{v}|} = \dfrac{d\mathbf{r}}{ds}$
- $\kappa = \left|\dfrac{d\mathbf{T}}{ds}\right| = \dfrac{1}{|\mathbf{v}|}\left|\dfrac{d\mathbf{T}}{dt}\right| = \dfrac{|\mathbf{v} \times \mathbf{a}|}{|\mathbf{v}|^3}$
- $\mathbf{N} = \dfrac{1}{\kappa}\dfrac{d\mathbf{T}}{ds} = \dfrac{d\mathbf{T}/dt}{|d\mathbf{T}/dt|}$

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