MATH 2551 J/HP Midterm 1
VERSION A
Spring 2026
COVERS SECTIONS 12.1-12.5, 13.1-13.4, 14.1-14.2

Full name: Key **GT ID:**

Honor code statement: I will abide strictly by the Georgia Tech honor code at all times. I will not use a calculator. **I do not have a phone within reach**, and I will not reference any website, application, or other CAS-enabled service. I will not consult with my notes or anyone during this exam. I will not provide aid to anyone else during this exam.

(   ) All of the knowledge presented in this exam is entirely my own. I am initialing to the left to attest to my integrity.

Read all instructions carefully before beginning.

- Print your name and GT ID neatly above.
- You have 75 minutes to take the exam.
- You may not use aids of any kind.
- Please show your work [J] and annotate your work using proper notation [N].
- Good luck!

Question	Points
1	2
2	2
3	4
4	8
5	8
6	10
7	8
8	8
Total:	50

For T/F problems choose whether the statement is true or false. If the statement is *always* true, pick true. If the statement is *ever* false, pick false. Also please be sure to neatly fill in the bubble corresponding to your answer choice. [A]

1. (2 points) If $\mathbf{u}$ and $\mathbf{v}$ are orthogonal unit vectors in $\mathbb{R}^3$, then $\|\mathbf{u} \times \mathbf{v}\| = 1$.

● TRUE ○ FALSE

from formula sheet

$\|u \times v\| = \|u\| \cdot \|v\| \, |\sin\theta| = 1 * 1 * \sin(\pi/2)$
$= 1$ ✓

2. (2 points) Which of the following vectors could be the principal unit normal vector at time $t = 2$ to a curve with parametrization $\mathbf{r}(t)$ whose tangent line at $t = 2$ is given by

$$\ell(s) = \langle 1, 1, 1\rangle + s\langle 2, 1, 2\rangle, \ s \in \mathbb{R}.$$

You do not need to justify your answers. *Select all that apply.* [A]

○ **A)** $\langle 2, -1, 2\rangle$ (A) not unit length ×

○ **B)** $\left\langle \frac{2}{3}, -\frac{1}{3}, \frac{2}{3}\right\rangle$

● **C)** $\left\langle \frac{1}{3}, \frac{2}{3}, -\frac{2}{3}\right\rangle$

● **D)** $\left\langle \frac{1}{\sqrt{5}}, -\frac{2}{\sqrt{5}}, 0\right\rangle$

○ **E)** $\left\langle \frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}\right\rangle$

(B) $v = \langle 2, 1, 2\rangle$ $u = \langle 2/3, -1/3, 2/3\rangle$
v not orthogonal to u ×

(C) $v = \langle 2, 1, 2\rangle$ $u = \langle 1/3, 2/3, -2/3\rangle$
$\|u\| = 1$ and $v \cdot u = \frac{2}{3} + \frac{2}{3} - \frac{4}{3} = 0$ ✓

(D) $v = \langle 2,1,2\rangle$ is orthogonal to $u = \langle 1/\sqrt{5}, \frac{-2}{\sqrt{5}}, 0\rangle$ and $\|u\| = 1$ so yes. ✓

(E) $v = \langle 2,1,2\rangle$ not orthogonal to $u = \langle 1/\sqrt{3}, 1/\sqrt{3}, 1/\sqrt{3}\rangle$ ×

3. (4 points) Find the area of the parallelogram determined by $P(1, 1, 1)$, $Q(2, 3, 1)$, $R(4, 2, 1)$, and $S(5, 4, 1)$. [AJN]

$\boxed{5}$

$\vec{PQ} = \langle 1, 2, 0\rangle$

$\vec{PR} = \langle 3, 1, 0\rangle$

So $\vec{PQ} \times \vec{PR} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 2 & 0 \\ 3 & 1 & 0 \end{vmatrix} = \langle 0, 0, 1-6\rangle$

$= \langle 0, 0, -5\rangle$

and area $= \|\vec{PQ} \times \vec{PR}\| = 5$

4. (8 points) Let P be the plane defined by $x - 2y + 3z = 25$. Find (a) the vector equation for the line ℓ passing through the point $Q(2, 0, 1)$ which is orthogonal to P, and (b) find the intersection between this line ℓ and the plane P.

[AJN]

(a) $\boxed{l(t) = \langle 2,0,1\rangle + t\langle 1,-2,3\rangle,\ t\in\mathbb{R}}$

(b) $\boxed{\left(\frac{24}{7}, \frac{-20}{7}, \frac{37}{7}\right)}$

(a)

$n = \langle 1,-2,3\rangle$

and $Q(2,0,1)$ so use equation.

$l(t) = \vec{OQ} + t\vec{n},\ t\in\mathbb{R}$

(b) $l(t) = \langle 2+t, -2t, 1+3t\rangle,\ t\in\mathbb{R}$, point on line is in plane if $x - 2y + 3z = 25$ so plug into plane eqn. & solve for t

$(2+t) - 2(-2t) + 3(1+3t) = 25$

$\Rightarrow\ 2 + t + 4t + 3 + 9t = 25$

$\Rightarrow\ 5 + 14t = 25 \Rightarrow 14t = 20 \Rightarrow t = 20/14 = 10/7$

So $l(\ ) = \langle 2,0,1\rangle + \frac{10}{7}\langle 1,-2,3\rangle$

$= \left\langle \frac{24}{7}, \frac{-20}{7}, \frac{37}{7}\right\rangle$

5. (10 points) Let $\mathbf{r}(t) = \langle 2\cos 2t, 2\sin 2t, 4t\rangle$, $0 \le t \le \pi$. Find (a) the curve's unit tangent vector $\mathbf{T}(t)$ and (b) the length of the curve parametrized by $\mathbf{r}(t)$. [AJN]

$\mathbf{T}(t) = \boxed{\langle -\frac{1}{\sqrt{2}}\sin 2t, \frac{1}{\sqrt{2}}\cos 2t, 1/\sqrt{2}\rangle}$

length is $\boxed{4\sqrt{2}\pi}$

(a)

$r'(t) = \langle -4\sin 2t, 4\cos 2t, 4\rangle$

$\|r'(t)\| = \sqrt{16\sin^2 2t + 16\cos^2 2t + 16}$

$= \sqrt{16+16} = \sqrt{32} = 4\sqrt{2}$

So $T(t) = \frac{1}{4\sqrt{2}}\langle -4\sin 2t, 4\cos 2t, 4\rangle = \langle -\frac{1}{\sqrt{2}}\sin 2t, \frac{1}{\sqrt{2}}\cos 2t, \frac{1}{\sqrt{2}}\rangle$

(b) $L = \int_0^{\pi} \|r'(t)\|\, dt = \int_0^{\pi} 4\sqrt{2}\, dt = 4\sqrt{2}\,t \Big|_0^{\pi}$

$= 4\sqrt{2}\pi - 0$

$= 4\sqrt{2}\pi$

6. (8 points) In this problem, you will work with the curve

$$\mathbf{r}(t) = t\mathbf{i} + (\ln \cos t)\mathbf{j}$$

for $-\pi/2 < t < \pi/2$. [AJN]

(a) Compute the principal unit normal vector $\mathbf{N}(t)$.

(b) Compute the curvature $\kappa(t)$.

$\mathbf{N}(t) = \boxed{\langle -\sin t, -\cos t \rangle}$

$\kappa(t) = \boxed{\cos t}$

$r'(t) = \langle 1, \frac{1}{\cos t} \cdot -\sin t \rangle$

$= \langle 1, -\tan t \rangle$

$\|r'(t)\| = \sqrt{1+\tan^2 t} = \sec t \quad (-\pi/2 < t < \pi/2)$

So $T(t) = \frac{r'(t)}{\|r'(t)\|} = \langle \frac{1}{\sec t}, \frac{-\tan t}{\sec t} \rangle = \langle \cos t, -\sin t \rangle$

Now $N(t) = \frac{T'(t)}{\|T'(t)\|}$, and $\kappa(t) = \frac{\|T'(t)\|}{\|r'(t)\|}$

$T'(t) = \langle -\sin t, -\cos t \rangle$ and $\|T'(t)\| = 1$.

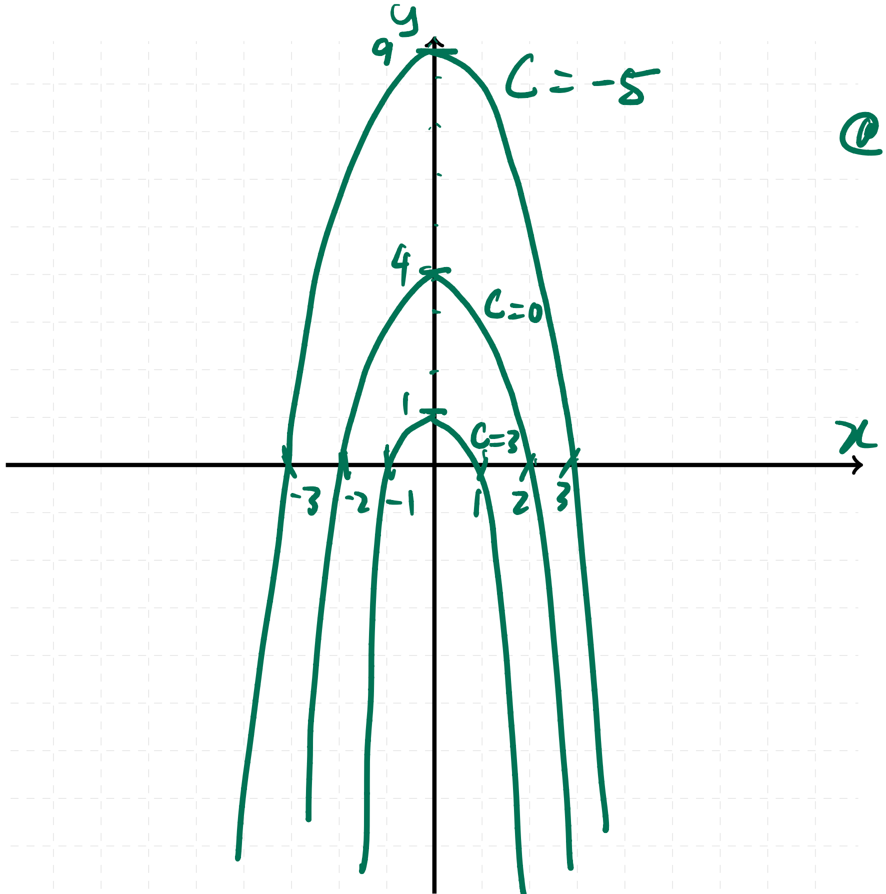
So $N(t) = \langle -\sin t, -\cos t \rangle$

and $\kappa(t) = \frac{1}{\sec t} = \cos t$

7. (8 points) Draw a contour map on the axes provided including all three of the level curves $g(x, y) = c$ for the function

$$g(x, y) = 4 - x^2 - y, \quad c = -5, 0, 3.$$

Show your work for how you find the equation of each level set, include labels for the axes, and label each level set as well as any x-intercepts or y-intercepts of each level set. [AJN]



@ $C=-5$ $g(x,y)=-5$

So

$4-x^2-y=-5$

$\Rightarrow y=-x^2+9$

y-intercept $(0,9)$

x-intercepts $(\pm 3, 0)$

@ $C=0$ $g(x,y)=0$

So

$4-x^2-y=0$

$\Rightarrow y=-x^2+4$

y-intercept $(0,4)$

x-intercepts $(\pm 2, 0)$

@ $C=3$ $g(x,y)=3$

So

$4-x^2-y=3$

$\Rightarrow y=-x^2+1$

y-intercept $(0,1)$

x-intercepts $(\pm 1, 0)$

8. (8 points) Show that the limit does not exist. To receive full credit, you must show work supporting your answer, use proper limit notation, and mention the test that you are using. [AJN]

$$\lim_{(x,y)\to(0,0)} \frac{x^2y}{x^4+xy^2}$$

Path 1:
along $y=x^2$

$$\lim_{(x,y)\to(0,0)} \frac{x^2y}{x^4+xy^2} = \lim_{(x,x^2)\to(0,0)} \frac{x^2x^2}{x^4+xx^4}$$

$$= \lim_{x\to0} \frac{x^4}{x^4+x^5} = \lim_{x\to0} \frac{1}{1+x} = 1.$$

Path 2:
along $y=2x$

$$\lim_{(x,y)\to(0,0)} \frac{x^2y}{x^4+xy^2} = \lim_{(x,x)\to(0,0)} \frac{2x^3}{x^4+4x^3}$$

$$= \lim_{x\to0} \frac{2}{x+4} = \frac{2}{4} = 1/2$$

The limit is $\boxed{\text{DNE by the TWO PATH TEST}}$

FORMULA SHEET

- $\langle u_1, u_2, u_3 \rangle \cdot \langle v_1, v_2, v_3 \rangle = u_1 v_1 + u_2 v_2 + u_3 v_3$
- $\mathbf{u} \cdot \mathbf{v} = |\mathbf{u}||\mathbf{v}| \cos(\theta)$
- $\langle u_1, u_2, u_3 \rangle \times \langle v_1, v_2, v_3 \rangle = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ u_1 & u_2 & u_3 \\ v_1 & v_2 & v_3 \end{vmatrix}$
- $|\mathbf{u} \times \mathbf{v}| = |\mathbf{u}||\mathbf{v}||\sin(\theta)|$
- $L = \displaystyle\int_a^b |\mathbf{r}'(t)| \, dt$
- $s(t) = \displaystyle\int_{t_0}^t |\mathbf{r}'(\tau)| \, d\tau$
- $\mathbf{T} = \dfrac{\mathbf{v}}{|\mathbf{v}|} = \dfrac{d\mathbf{r}}{ds}$
- $\kappa = \left|\dfrac{d\mathbf{T}}{ds}\right| = \dfrac{1}{|\mathbf{v}|}\left|\dfrac{d\mathbf{T}}{dt}\right| = \dfrac{|\mathbf{v} \times \mathbf{a}|}{|\mathbf{v}|^3}$
- $\mathbf{N} = \dfrac{1}{\kappa}\dfrac{d\mathbf{T}}{ds} = \dfrac{d\mathbf{T}/dt}{|d\mathbf{T}/dt|}$

SCRATCH PAPER - PAGE LEFT BLANK