MATH 2551 J/HP Midterm 1 Make-up
VERSION C
Spring 2026
COVERS SECTIONS 12.1-12.5, 13.1-13.4, 14.1-14.2

Full name: ⎯⎯⎯⎯⎯⎯⎯⎯⎯⎯⎯⎯⎯⎯⎯⎯⎯⎯⎯⎯⎯ **GT ID:**⎯⎯⎯⎯⎯⎯⎯

Honor code statement: I will abide strictly by the Georgia Tech honor code at all times. I will not use a calculator. **I do not have a phone within reach**, and I will not reference any website, application, or other CAS-enabled service. I will not consult with my notes or anyone during this exam. I will not provide aid to anyone else during this exam.

(   ) All of the knowledge presented in this exam is entirely my own. I am initialing to the left to attest to my integrity.

Read all instructions carefully before beginning.

- Print your name and GT ID neatly above.

- You have 75 minutes to take the exam.

- You may not use aids of any kind.

- Please show your work [J] and annotate your work using proper notation [N].

- Good luck!

Question	Points
1	2
2	2
3	4
4	8
5	10
6	10
7	6
8	8
Total:	50

For T/F problems choose whether the statement is true or false. If the statement is *always* true, pick true. If the statement is *ever* false, pick false. Also please be sure to neatly fill in the bubble corresponding to your answer choice. [A]

1. (2 points) If $\mathbf{n} = \mathbf{u} \times \mathbf{v}$, for some vectors $\mathbf{u}, \mathbf{v}$ in $\mathbb{R}^3$, then $(\mathbf{u} \cdot \mathbf{n})^2 + (\mathbf{v} \cdot \mathbf{n})^2 = 0$.

   ○ **TRUE** ○ **FALSE**

2. (2 points) The curvature of a circle of radius 16 is larger than the curvature of a circle of radius 4.

   ○ **TRUE** ○ **FALSE**

3. (4 points) Consider the line L_1 parametrized by $\mathbf{r}_1(t) = \langle t + 2, 2t - 1, t \rangle$, $t \in \mathbb{R}$, and the line L_2 parametrized by $\mathbf{r}_2(t) = \langle -t + 2, 2t - 1, 3t \rangle$, $t \in \mathbb{R}$. Find an equation of the plane which contains both L_1 and L_2. [AJN]

4. (8 points) Let $P(1,1,1)$ and $Q(2,3,0)$. Find (a) the vector equation for the line ℓ passing through the points P and Q, and (b) find the intersection between this line ℓ and the plane $2x - y + z = 5$.

[AJN]

(a)

(b)

5. (10 points) Let $\mathbf{r}(t) = \langle e^t \cos t, e^t \sin t, 2 \rangle$, $t \in \mathbb{R}$. Find the curve's unit tangent vector $\mathbf{T}(t)$, the principal unit normal vector $\mathbf{N}(t)$, and the curvature $\kappa(t)$. 　　　　　　[AJN]

$\mathbf{T}(t) =$

$\mathbf{N}(t) =$

$\kappa(t) =$

6. (10 points) In this problem, you will work with the curve   [AJN]

$$\mathbf{r}(t) = \langle 3\sin t, 5\cos t, 4\sin t \rangle, \ 0 \le t < 2\pi.$$

(a) Compute the arc length function $s(t)$, taking $t_0 = 0$ as your base point.

(b) Compute the distance along C between the points $(3, 0, 4)$ and $(0, -5, 0)$.

(c) Find an arc-length parametrization for the curve C.

Note: box your answers.

7. (6 points) Let $f(x, y) = \frac{1}{\sqrt{9 - x^2 - y}}$. Graph the domain of f on the provided axes below, and clearly label the axes and all intercepts. Be sure to show all your work in finding the domain. Indicate whether or not each part of the boundary of the domain is included. [AJN]
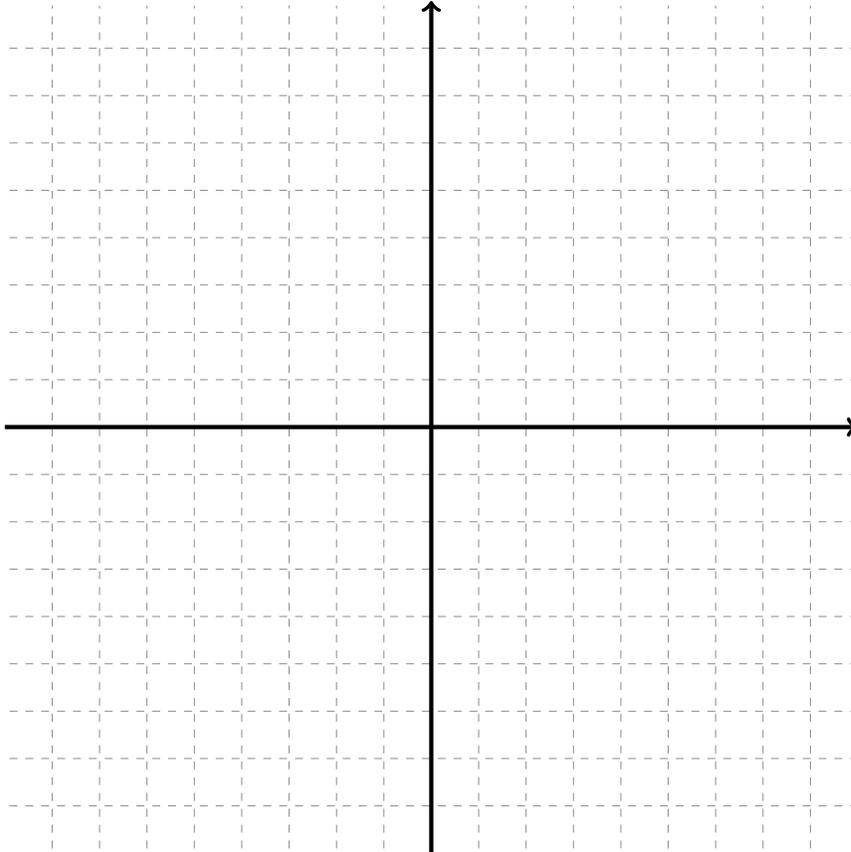
8. (8 points) Show that the limit does not exist. To receive full credit, you must show work supporting your answer, use proper limit notation, and mention the test that you are using. [AJN]

$$\lim_{(x,y)\to(1,1)} \frac{x-y}{x+y-2}$$

FORMULA SHEET

- $\langle u_1, u_2, u_3 \rangle \cdot \langle v_1, v_2, v_3 \rangle = u_1 v_1 + u_2 v_2 + u_3 v_3$

- $\mathbf{u} \cdot \mathbf{v} = |\mathbf{u}||\mathbf{v}| \cos(\theta)$

- $\langle u_1, u_2, u_3 \rangle \times \langle v_1, v_2, v_3 \rangle = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ u_1 & u_2 & u_3 \\ v_1 & v_2 & v_3 \end{vmatrix}$

- $|\mathbf{u} \times \mathbf{v}| = |\mathbf{u}||\mathbf{v}||\sin(\theta)|$

- $L = \displaystyle\int_a^b |\mathbf{r}'(t)| \; dt$

- $s(t) = \displaystyle\int_{t_0}^t |\mathbf{r}'(\tau)| \; d\tau$

- $\mathbf{T} = \dfrac{\mathbf{v}}{|\mathbf{v}|} = \dfrac{d\mathbf{r}}{ds}$

- $\kappa = \left| \dfrac{d\mathbf{T}}{ds} \right| = \dfrac{1}{|\mathbf{v}|} \left| \dfrac{d\mathbf{T}}{dt} \right| = \dfrac{|\mathbf{v} \times \mathbf{a}|}{|\mathbf{v}|^3}$

- $\mathbf{N} = \dfrac{1}{\kappa} \dfrac{d\mathbf{T}}{ds} = \dfrac{d\mathbf{T}/dt}{|d\mathbf{T}/dt|}$

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