MATH 2551 J/HP Midterm 2 Make-up
VERSION C
Spring 2026
COVERS SECTIONS 12.1-12.5, 13.1-13.4, 14.1-14.2

Full name: ______________________________ **GT ID:**____________

Honor code statement: I will abide strictly by the Georgia Tech honor code at all times. I will not use a calculator. **I do not have a phone within reach**, and I will not reference any website, application, or other CAS-enabled service. I will not consult with my notes or anyone during this exam. I will not provide aid to anyone else during this exam.

(   ) All of the knowledge presented in this exam is entirely my own. I am initialing to the left to attest to my integrity.

Read all instructions carefully before beginning.

- Print your name and GT ID neatly above.
- You have 75 minutes to take the exam.
- You may not use aids of any kind.
- Please show your work [J] and annotate your work using proper notation [N].
- Good luck!

Question	Points
1	2
2	4
3	5
4	8
5	8
6	8
7	8
8	8
Total:	51

For T/F problems choose whether each statement is true or false. If the statement is *always* true, pick true. If the statement is *ever* false, pick false. For all problems on this page please be sure to neatly fill in the bubble corresponding to your answer choice. [A]

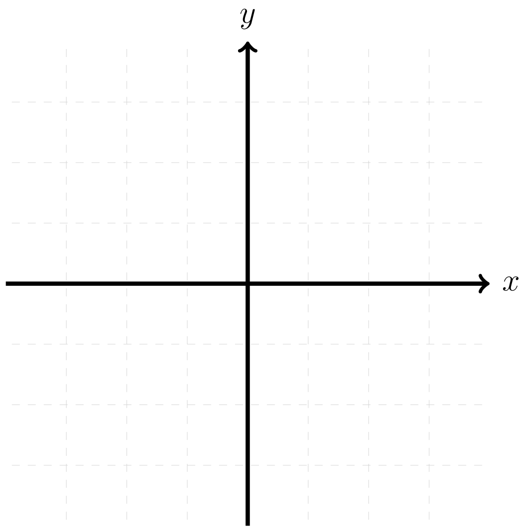
1. (2 points) If $f(x,y) = \sin(xy)$, then the direction such that the function f increases most rapidly at the point $(0,1)$ is parallel to the y-axis.

   ◯ **TRUE** ◯ **FALSE**

2. (4 points) Suppose $f(x,y) = e^{-(x^2+y^2)}$. Find the equation for the plane that is tangent to the surface of $z = f(x,y)$ at the point $(0,0,1)$. [AJN]

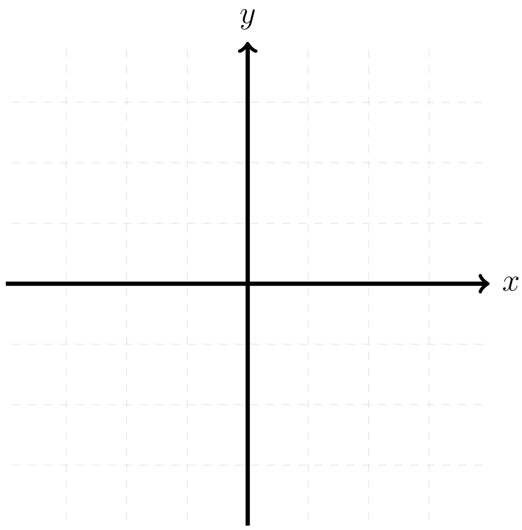
3. (5 points) Sketch R the region of integration and switch the order of integration. That is, rewrite the integral in the order $dy\,dx$. *Do not evaluate!* [AN]

$$\int_0^2 \int_{y^2}^4 \sin(x^3)\;dx\;dy$$



4. (8 points) Sketch R the region of integration and (a) set up an integral in polar coordinates for evaluating $\iint_R f(x,y)\,dA$. Note that R is the region outside the circle of radius $r_1 = 1$ and inside the circle of radius $r_2 = 2$ which is to the left of the y-axis. Then, (b) evaluate the double integral. [AN]

$$R = \left\{(x,y) \in \mathbb{R}^2 \mid 1 \le x^2 + y^2 \le 4,\ x \le 0\right\},\ \text{and } f(x,y) = \frac{x}{\sqrt{x^2+y^2}}.$$



(a)

(b)

5. (7 points) Use the function $z = f(x, y)$ and the point P to answer the questions below. [AJN]

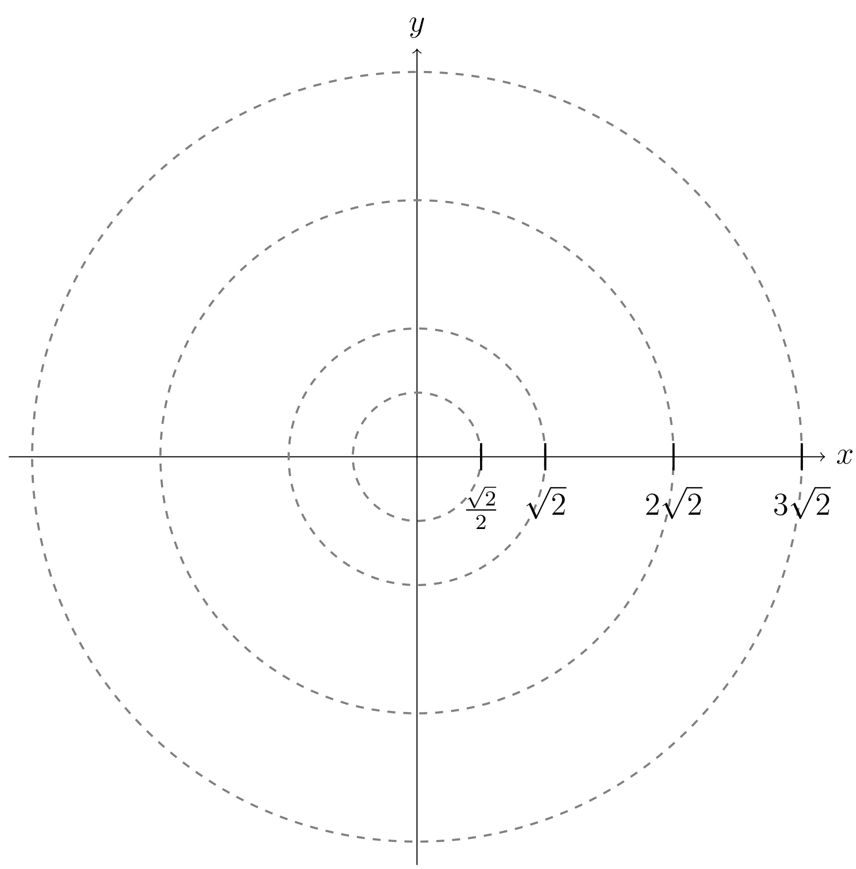
$$f(x, y) = 2\ln(x^2 + y^2) \text{ and } P(-2, 2)$$

(a) Find the gradient ∇f of f.

(b) Find the directional derivative $D_{\mathbf{u}}f$ of f in the direction of $\mathbf{u} = \langle 1, 3\rangle$ at $P(-2, 2)$.

(c) Find a unit vector $\mathbf{v}$ which points in the direction which *maximizes* the value of $D_{\mathbf{u}}f$ at $P(-2, 2)$.

(d) Sketch $P(-2, 2)$ and the gradient vector $\nabla f(P)$ on the axes provided.



6. (8 points) Let $f : \mathbb{R}^3 \to \mathbb{R}$ be the function $f(x, y) = x^2 e^y$ and $\mathbf{r}(s, t) : \mathbb{R}^2 \to \mathbb{R}^3$ be the function

$$\mathbf{r}(s,t) = \begin{bmatrix} s/t \\ s+t \end{bmatrix}.$$

   (a) Find Df.

   (b) Find $D\mathbf{r}$.

   (c) Evaluate $\mathbf{r}(1, 2)$ and $D\mathbf{r}|_{(s,t)=(1,2)}$.

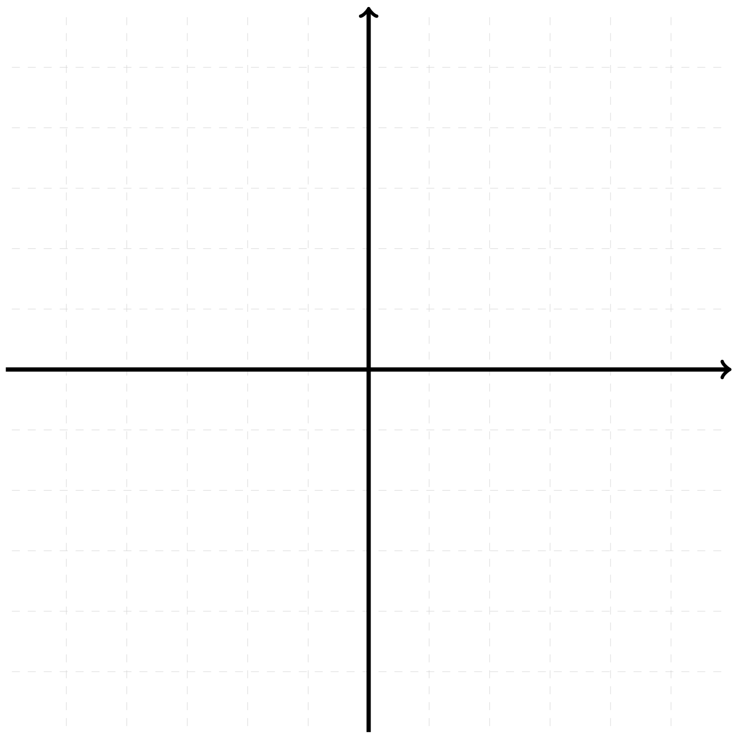
   (d) Finally, use the Chain Rule to evaluate $D(f(\mathbf{r}(s, t)))|_{(s,t)=(1,2)}$.
   Please put the answer to part (d) in the box below.

[AJN]

(d)

7. (8 points) Sketch the region R including all labels for each of the axes, each boundary curve, and each intersection point between curves and axes. Then, compute the value of $\iint_R f(x,y)\, dA$ where $f(x,y) = y^5 e^{xy^2}$ and R is the region in the first quadrant bounded by the lines $x = 0$, $y = \sqrt{x}$, and $y = 2$. *Hint: use the order $dx\, dy$ in the iterated integral.*

[AJN]



8. (8 points) Use the method of Lagrange Multipliers to find the maximum and minimum value of $f(x, y) = xe^{\sqrt{6}\,y}$ subject to the constraint $x^2 + y^2 = 1$. [AJN]

FORMULA SHEET

- Total Derivative: For $\mathbf{f}(x_1,\ldots,x_n) = \langle f_1(x_1,\ldots,x_n),\ldots,f_m(x_1,\ldots,x_n)\rangle$

$$D\mathbf{f} = \begin{bmatrix} (f_1)_{x_1} & (f_1)_{x_2} & \cdots & (f_1)_{x_n} \\ (f_2)_{x_1} & (f_2)_{x_2} & \cdots & (f_2)_{x_n} \\ \vdots & \ddots & \cdots & \vdots \\ (f_m)_{x_1} & (f_m)_{x_2} & \cdots & (f_m)_{x_n} \end{bmatrix}$$

- Linearization: Near $\mathbf{a}$, $f(\mathbf{x}) \approx L(\mathbf{x}) = f(\mathbf{a}) + Df(\mathbf{a})(\mathbf{x}-\mathbf{a})$

- Chain Rule: If $h = g(f(\mathbf{x}))$ then $Dh(\mathbf{x}) = Dg(f(\mathbf{x}))Df(\mathbf{x})$

- Implicit Differentiation: If z is implicitly given in terms of x and y by $F(x,y,z) = c$, then $\dfrac{\partial z}{\partial x} = \dfrac{-F_x}{F_z}$ and $\dfrac{\partial z}{\partial y} = \dfrac{-F_y}{F_z}$.

- Directional Derivative: If $\mathbf{u}$ is a unit vector, $D_{\mathbf{u}}f(P) = Df(P)\mathbf{u} = \nabla f(P)\cdot\mathbf{u}$

- The tangent line to a level curve of $f(x,y)$ at (a,b) is $0 = \nabla f(a,b)\cdot\langle x-a, y-b\rangle$

- The tangent plane to a level surface of $f(x,y,z)$ at (a,b,c) is

$$0 = \nabla f(a,b,c)\cdot\langle x-a, y-b, z-c\rangle.$$

- Hessian Matrix: For $f(x,y)$, $Hf(x,y) = \begin{bmatrix} f_{xx} & f_{yx} \\ f_{xy} & f_{yy}\end{bmatrix}$

- Second Derivative Test: If (a,b) is a critical point of $f(x,y)$ then
  1. If $\det(Hf(a,b)) > 0$ and $f_{xx}(a,b) < 0$ then f has a local maximum at (a,b)
  2. If $\det(Hf(a,b)) > 0$ and $f_{xx}(a,b) > 0$ then f has a local minimum at (a,b)
  3. If $\det(Hf(a,b)) < 0$ then f has a saddle point at (a,b)
  4. If $\det(Hf(a,b)) = 0$ the test is inconclusive

- Area/volume: $\text{area}(R) = \iint_R dA$

- Trig identities: $\sin^2(x) = \frac{1}{2}(1-\cos(2x))$, $\cos^2(x) = \frac{1}{2}(1+\cos(2x))$

- Average value: $f_{avg} = \dfrac{\iint_R f(x,y)dA}{\text{area of } R}$

- Polar coordinates: $x = r\cos(\theta)$, $y = r\sin(\theta)$, $dA = r\ dr\ d\theta$

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