

**MATH 2551 Midterm 3**  
**VERSION A**  
**Spring 2026**  
**COVERS SECTIONS 15.5-15.8, 16.1-16.8**

**Full name:** \_\_\_\_\_ **GT ID:** \_\_\_\_\_

**Honor code statement:** I will abide strictly by the Georgia Tech honor code at all times. I will not use a calculator. I do not have a phone within reach, and I will not reference any website, application, or other CAS-enabled service. I will not consult with my notes or anyone during this exam. I will not provide aid to anyone else during this exam.

(     ) All of the knowledge presented in this exam is entirely my own. I am initialing to the left to attest to my integrity.

**Read all instructions carefully** before beginning.

- Print your name and GT ID neatly above.
- You have 75 minutes to take the exam.
- You may not use aids of any kind.
- Please show your work [J] and annotate your work using proper notation [N].
- Good luck!

Question	Points
1	2
2	2
3	6
4	10
5	10
6	10
7	10
Total:	50

For problems 1-2 choose whether each statement is true or false. If the statement is *always* true, pick true. If the statement is *ever* false, pick false. For all problems on this page please be sure to neatly fill in the bubble corresponding to your answer choice. [A]

1. (2 points) If  $C_1$  and  $C_2$  are two curves with the same starting point and ending point, then  $\int_{C_1} \nabla f \cdot \mathbf{T} ds = \int_{C_2} \nabla f \cdot \mathbf{T} ds$ .

- TRUE       FALSE

2. (2 points) If  $C$  is a curve with parametrization  $\mathbf{r}_1(t) = \langle x(t), y(t) \rangle$  with  $t \in [a, b]$ , then the parametrization  $\mathbf{r}_2(t) = \mathbf{r}_1(-t)$  with  $t \in [b, a]$  is also a parametrization of  $C$  with the opposite orientation. [A]

- TRUE       FALSE

3. (6 points) Let  $S$  be the surface which is the cap of a sphere of radius 2 units with  $z \geq \sqrt{3}$ . The point  $P(1, 0, \sqrt{3})$  is on  $S$  at the intersection of the sphere and the plane. [AN]

- (a) Sketch the point  $P$  on the image provided below.
- (b) Find the spherical coordinates  $(\rho, \varphi, \theta)$  of the point  $P(1, 0, \sqrt{3})$ .
- (c) Find a parametrization  $r(u, v)$  of  $S$ . *Hint: spherical coordinates.*

(a)

(b) =

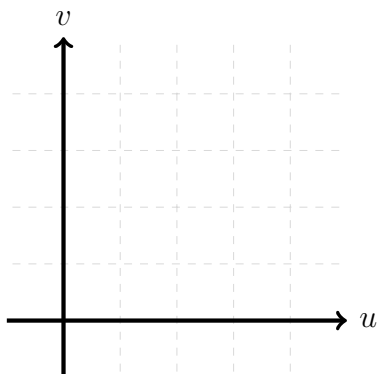
(c)

4. (10 points) In this problem you will compute the integral

$$M = \iint_R (3x + y)(x - y) \, dx \, dy$$

for the region  $R$  bounded by the lines  $y = -3x$ ,  $y = -3x + 2$ ,  $y = x$ ,  $y = x - 3$  using the transformation  $u = 3x + y$ ,  $v = x - y$ . [AJN]

- On the axes provided, sketch the new region of integration  $G$  in the  $uv$ -plane.
- Solve the transformation equations for  $x$  and  $y$  in terms of  $u$  and  $v$  and compute the Jacobian determinant, i.e., find  $\mathbf{T}(u, v)$  and  $|\det D\mathbf{T}(u, v)|$ .
- Evaluate the double integral using a change of variables and your results from parts (a) and (b) to find the value of  $M$ .

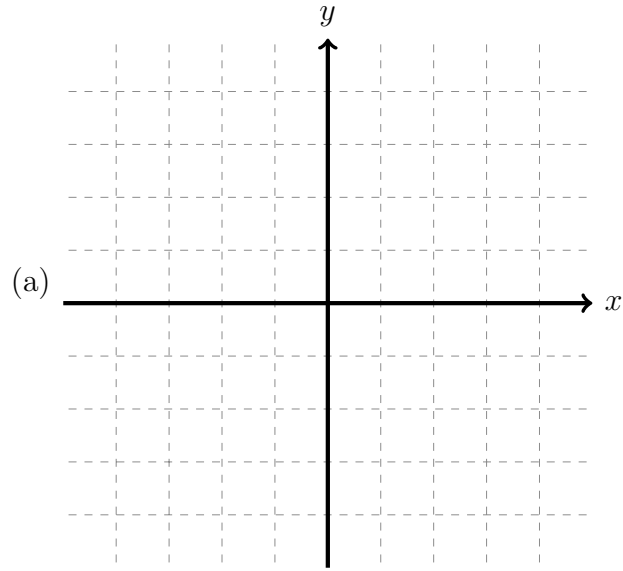
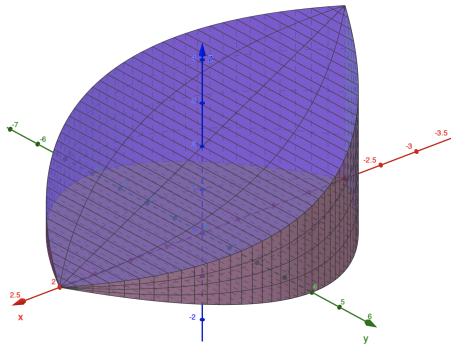


$\mathbf{T}(u, v) =$

$|\det D\mathbf{T}(u, v)| =$

5. (10 points) In this problem you will compute the mass of the solid  $D$  pictured below, which occupies the region which is bounded below by  $z = 0$ , bounded above by the plane  $z = 2 - x$ , and with base  $R$  in the  $xy$ -plane between the curves  $y = 4 - x^2$  and  $y = x^2 - 4$ . [AJN]

- (a) Sketch the region  $R$  bounded by curves  $y = 4 - x^2$  and  $y = x^2 - 4$  in the  $xy$ -plane. Determine the points of intersection of the two curves.
- (b) Find the volume of  $D$  bounded by the surfaces  $y = 4 - x^2$ ,  $y = x^2 - 4$ ,  $z = 0$  and  $z = 2 - x$  in 3-dimensional space.

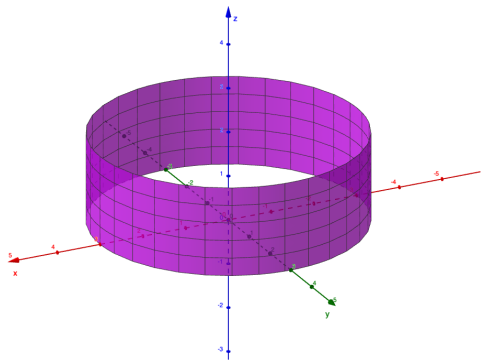


(b)

6. (10 points) This problem will have you compute the flux of the vector field  $\mathbf{F} = \langle x, y, z^2 \rangle$  through  $S$  the open ended circular cylinder of radius 3 and height 2 with its base on the  $xy$ -plane and centered about the  $z$ -axis, oriented away from the  $z$ -axis. [AJN]

(a) Find a parametrization  $r(u, v)$  of the surface  $S$ . *Hint: cylindrical coordinates.*

(b) Set up and **evaluate** a surface integral which computes the flux of  $\mathbf{F}$  through  $S$ .



(a)

(b)

7. (10 points) Consider the vector field  $\mathbf{F} = \langle -y, x \rangle$  and the curve  $C$  which is the circle  $x^2 + y^2 = 4$  oriented counterclockwise with outward pointing normal vector. [AJN]
- (a) Compute the flow of  $\mathbf{F}$  around  $C$ , which is  $\int_C \mathbf{F} \cdot \mathbf{T} ds$ , **with or without** using Green's Theorem. *Use either method, but you can use the other integral to check your answer.*
- (b) Compute the flux of  $F$  around  $C$ , which is  $\int_C \mathbf{F} \cdot \mathbf{n} ds$ , **using** Green's Theorem.
- (c) Is  $F$  conservative? Explain in at least one or two complete sentences how you know.

(a)

(b)

### FORMULA SHEET

- Trig identities:  $\sin^2(x) = \frac{1}{2}(1 - \cos(2x))$ ,  $\cos^2(x) = \frac{1}{2}(1 + \cos(2x))$
- Volume( $D$ ) =  $\iiint_D dV$ ,  $f_{avg} = \frac{\iiint_D f(x, y, z) dV}{\text{volume of } D}$  or  $\frac{\int_C f(x, y, z) ds}{\text{length of } C}$ ,  
Mass:  $M = \iiint_D \delta dV$
- Cylindrical coordinates:  $x = r \cos(\theta)$ ,  $y = r \sin(\theta)$ ,  $z = z$ ,  $dV = r dz dr d\theta$
- Spherical coordinates:  $x = \rho \sin(\phi) \cos(\theta)$ ,  $y = \rho \sin(\phi) \sin(\theta)$ ,  $z = \rho \cos(\phi)$ ,  
 $dV = \rho^2 \sin(\phi) d\rho d\phi d\theta$
- Substitution for double integrals: If  $R$  is the image of  $G$  under a coordinate transformation  $\mathbf{T}(u, v) = \langle x(u, v), y(u, v) \rangle$  then

$$\iint_R f(x, y) dx dy = \iint_G f(\mathbf{T}(u, v)) |\det D\mathbf{T}(u, v)| du dv.$$

- Scalar line integral:  $\int_C f(x, y, z) ds = \int_a^b f(\mathbf{r}(t)) |\mathbf{r}'(t)| dt$
- Tangential vector line integral:  $\int_C \mathbf{F} \cdot \mathbf{T} ds = \int_C \mathbf{F} \cdot d\mathbf{r} = \int_a^b \mathbf{F}(\mathbf{r}(t)) \cdot \mathbf{r}'(t) dt$
- Normal vector line integral:  $\int_C \mathbf{F}(x, y) \cdot \mathbf{n} ds = \int_C P dy - Q dx = \int_a^b \mathbf{F}(\mathbf{r}(t)) \cdot \langle y'(t), -x'(t) \rangle dt$ .
- Fundamental Theorem of Line Integrals:  $\int_C \nabla f \cdot d\mathbf{r} = f(B) - f(A)$  if  $C$  is a path from  $A$  to  $B$
- Curl (Mixed Partials) Test:  $\mathbf{F} = \nabla f$  if  $\text{curl } \mathbf{F} = \mathbf{0} \Leftrightarrow P_z = R_x, Q_z = R_y$ , and  $Q_x = P_y$ .
- $\nabla = \left\langle \frac{\partial}{\partial x}, \frac{\partial}{\partial y}, \frac{\partial}{\partial z} \right\rangle$        $\text{div } \mathbf{F} = \nabla \cdot \mathbf{F}$        $\text{curl } \mathbf{F} = \nabla \times \mathbf{F}$
- Green's Theorem: If  $C$  is a simple closed curve with positive orientation and  $R$  is the simply-connected region it encloses, then

$$\int_C \mathbf{F} \cdot \mathbf{T} ds = \iint_R (\nabla \times \mathbf{F}) \cdot \mathbf{k} dA \qquad \int_C \mathbf{F} \cdot \mathbf{n} ds = \iint_R (\nabla \cdot \mathbf{F}) dA.$$

- Surface Area =  $\iint_S 1 d\sigma$
- Scalar surface integral:  $\iint_S f(x, y, z) d\sigma = \iint_R f(\mathbf{r}(u, v)) |\mathbf{r}_u \times \mathbf{r}_v| dA$
- Flux surface integral:  $\iint_S \mathbf{F} \cdot \mathbf{n} d\sigma = \iint_S \mathbf{F} \cdot d\boldsymbol{\sigma} = \iint_R \mathbf{F}(\mathbf{r}(u, v)) \cdot (\mathbf{r}_u \times \mathbf{r}_v) dA$
- Stokes' Theorem: If  $S$  is a piecewise smooth oriented surface bounded by a piecewise smooth curve  $C$  and  $\mathbf{F}$  is a vector field whose components have continuous partial derivatives on an open region containing  $S$ , then

$$\int_C \mathbf{F} \cdot \mathbf{T} ds = \iint_S (\nabla \times \mathbf{F}) \cdot \mathbf{n} d\sigma.$$

- Divergence Theorem: If  $S$  is a piecewise smooth closed oriented surface enclosing a volume  $D$  and  $\mathbf{F}$  is a vector field whose components have continuous partial derivatives on  $D$ , then

$$\iint_S \mathbf{F} \cdot \mathbf{n} d\sigma = \iiint_D \nabla \cdot \mathbf{F} dV.$$

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