MATH 2551 Midterm 3 Make-up
VERSION C
Spring 2026
COVERS SECTIONS 15.5-15.8, 16.1-16.8

Full name: ______________________________ **GT ID:**____________

Honor code statement: I will abide strictly by the Georgia Tech honor code at all times. I will not use a calculator. I do not have a phone within reach, and I will not reference any website, application, or other CAS-enabled service. I will not consult with my notes or anyone during this exam. I will not provide aid to anyone else during this exam.

(   ) All of the knowledge presented in this exam is entirely my own. I am initialing to the left to attest to my integrity.

Read all instructions carefully before beginning.

- Print your name and GT ID neatly above.
- You have 75 minutes to take the exam.
- You may not use aids of any kind.
- Please show your work [J] and annotate your work using proper notation [N].
- Good luck!

Question	Points
1	2
2	2
3	6
4	10
5	10
6	10
7	10
Total:	50

For problems 1-2 choose whether each statement is true or false. If the statement is *always* true, pick true. If the statement is *ever* false, pick false. For all problems on this page please be sure to neatly fill in the bubble corresponding to your answer choice. [A]

1. (2 points) If $\mathbf{F} = \langle x - y, x^2 \rangle$ and C is the boundary of the rectangle with vertices $(0,0), (0,2), (1,0), (1,2)$ oriented counterclockwise, then

$$\int_C \mathbf{F} \cdot \mathbf{T}\, ds = \int_0^2 \int_0^1 (1 + 2y)\, dy dx$$

◯ **TRUE** ◯ **FALSE**

2. (2 points) The vector field $\mathbf{F}(x, y) = \langle 1, x \rangle$ is conservative.

◯ **TRUE** ◯ **FALSE**

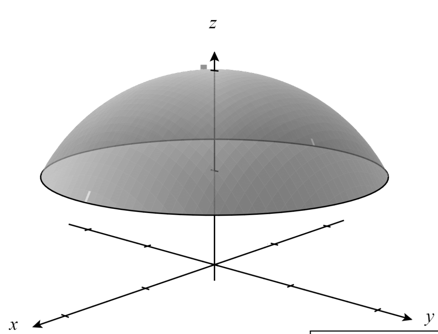
3. (6 points) Let S be the surface which is the cap of a sphere of radius 2 units with $z \geq 1$. The point $P(0, \sqrt{3}, 1)$ is on S at the intersection of the sphere and the plane. [AN]

   (a) Sketch the point P on the image provided below.

   (b) Find the spherical coordinates (ρ, φ, θ) of the point $P(0, \sqrt{3}, 1)$.

   (c) Find a parametrization $r(u, v)$ of S. *Hint: spherical coordinates.*

(a)



(b) =

(c)

4. (10 points) In this problem you will compute the integral

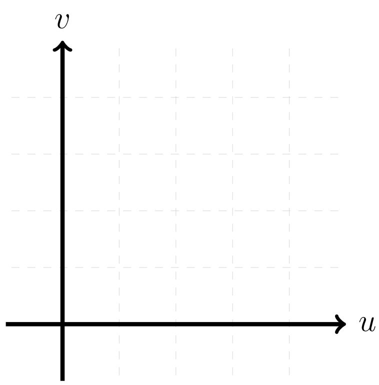
$$M = \iint_R (5x + 3y)(x + y)\, dx\, dy$$

for the region R bounded by the lines $y = -\frac{5}{3}x + \frac{1}{3}$, $y = -\frac{5}{3}x + 1$, $y = -x$, $y = -x + 4$ using the transformation $u = 5x + 3y$, $v = x + y$. [AJN]

(a) On the axes provided, sketch the new region of integration G in the uv-plane.

(b) Solve the transformation equations for x and y in terms of u and v and compute the Jacobian determinant, i.e., find $\mathbf{T}(u, v)$ and $|\det D\mathbf{T}(u, v)|$.

(c) Evaluate the double integral using a change of variables and your results from parts (a) and (b) to find the value of M.



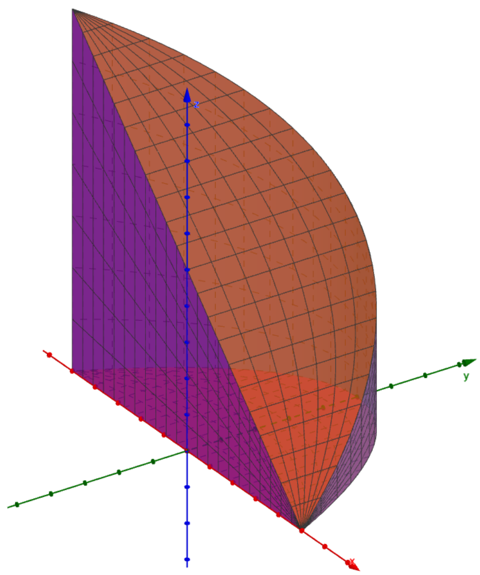
$\mathbf{T}(u, v)=$

$|\det D\mathbf{T}(u, v)|=$

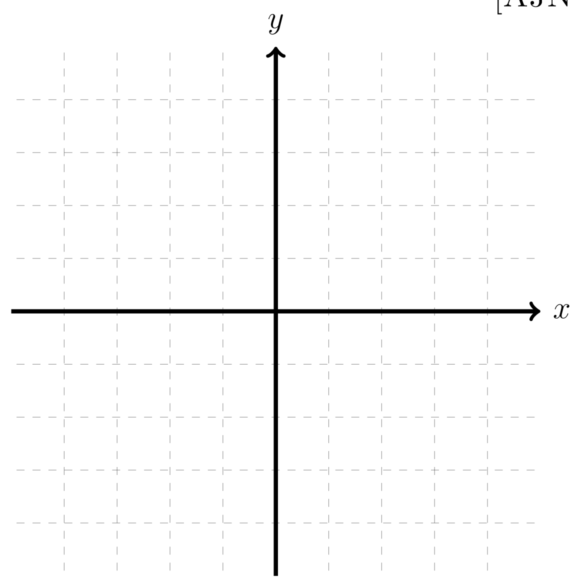
5. (10 points) In this problem you will compute the volume of the solid D pictured below, which occupies the region which is bounded below by $z = 0$, bounded above by the plane $z = 1 - x$, and with base R in the xy-plane between the curves $y = 1 - x^2$ and $y = 0$.

   (a) Sketch the region R bounded by curves $y = 1 - x^2$ and $y = 0$ in the xy-plane. Determine the points of intersection of the two curves.

   (b) Find the volume of D bounded by the surfaces $y = 1 - x^2$, $y = 0$, $z = 0$ and $z = 1 - x$ in 3-dimensional space. [AJN]



(a)



(b)

6. (10 points) Let $\mathbf{F}$ be the vector field given by

$$\mathbf{F}(x, y, z) = \langle x^2 + y - z, y^2 - x + z, x - y + z^3 \rangle.$$

Let C be the boundary of the square in the xy-coordinate plane in $\mathbb{R}^3$ defined by the points

$$(0, 0, 0), (0, 1, 0), (1, 0, 0), (1, 1, 0),$$

oriented counterclockwise when viewed from above. Calculate the value of $\int_C \mathbf{F} \cdot d\mathbf{r}$ the circulation of $\mathbf{F}$ around C. *Hint: Stokes' Theorem.* [AJN]

7. (10 points) The vector field $\mathbf{F} = \langle 2x+y^3, 3xy^2 \rangle$ is a conservative vector field. Let C which is the boundary of the square with vertices $(0,0), (0,1), (1,0), (1,1)$ oriented counterclockwise with outward pointing normal vector. [AJN]

   (a) Find a potential function for $\mathbf{F}$.

   (b) Compute the flow of $\mathbf{F}$ around C, which is $\int_C \mathbf{F} \cdot \mathbf{T}\, ds$, **using** Green's Theorem.

   (c) Compute the flux of F around C, which is $\int_C \mathbf{F} \cdot \mathbf{n}\, ds$, **with or without** using Green's Theorem. *Use either method, but you can use the other integral to check your answer.*

(a)

(b)

(c)

FORMULA SHEET

- Trig identities: $\sin^2(x) = \frac{1}{2}(1-\cos(2x))$, $\cos^2(x) = \frac{1}{2}(1+\cos(2x))$
- Volume$(D) = \iiint_D dV$, $\quad f_{avg} = \dfrac{\iiint_D f(x,y,z)dV}{\text{volume of } D}$ or $\dfrac{\int_C f(x,y,z)\ ds}{\text{length of } C}$,
  Mass: $M = \iiint_D \delta\ dV$
- Cylindrical coordinates: $x = r\cos(\theta)$, $\quad y = r\sin(\theta)$, $z = z$, $dV = r\ dz\ dr\ d\theta$
- Spherical coordinates: $x = \rho\sin(\phi)\cos(\theta)$, $y = \rho\sin(\phi)\sin(\theta)$, $z = \rho\cos(\phi)$,
  $dV = \rho^2\sin(\phi)\ d\rho\ d\phi\ d\theta$
- Substitution for double integrals: If R is the image of G under a coordinate transformation $\mathbf{T}(u,v) = \langle x(u,v), y(u,v)\rangle$ then

$$\iint_R f(x,y)\ dx\ dy = \iint_G f(\mathbf{T}(u,v))|\det D\mathbf{T}(u,v)|\ du\ dv.$$

- Scalar line integral: $\int_C f(x,y,z)\ ds = \int_a^b f(\mathbf{r}(t))|\mathbf{r}'(t)|dt$
- Tangential vector line integral: $\int_C \mathbf{F}\cdot\mathbf{T}\ ds = \int_C \mathbf{F}\cdot d\mathbf{r} = \int_a^b \mathbf{F}(\mathbf{r}(t))\cdot\mathbf{r}'(t)\ dt$
- Normal vector line integral: $\int_C \mathbf{F}(x,y)\cdot\mathbf{n}\ ds = \int_C P\ dy - Q\ dx = \int_a^b \mathbf{F}(\mathbf{r}(t))\cdot \langle y'(t), -x'(t)\rangle\ dt$.
- Fundamental Theorem of Line Integrals: $\int_C \nabla f\cdot d\mathbf{r} = f(B) - f(A)$ if C is a path from A to B
- Curl (Mixed Partials) Test: $\mathbf{F} = \nabla f$ if curl $\mathbf{F} = \mathbf{0} \Leftrightarrow P_z = R_x, Q_z = R_y$, and $Q_x = P_y$.
- $\nabla = \left\langle \dfrac{\partial}{\partial x}, \dfrac{\partial}{\partial y}, \dfrac{\partial}{\partial z}\right\rangle$ $\qquad$ div $\mathbf{F} = \nabla\cdot\mathbf{F}$ $\qquad$ curl $\mathbf{F} = \nabla\times\mathbf{F}$
- Green's Theorem: If C is a simple closed curve with positive orientation and R is the simply-connected region it encloses, then

$$\int_C \mathbf{F}\cdot\mathbf{T}\ ds = \iint_R (\nabla\times\mathbf{F})\cdot\mathbf{k}\ dA \qquad \int_C \mathbf{F}\cdot\mathbf{n}\ ds = \iint_R (\nabla\cdot\mathbf{F})\ dA.$$

- Surface Area$=\iint_S 1\ d\sigma$
- Scalar surface integral: $\iint_S f(x,y,z)\ d\sigma = \iint_R f(\mathbf{r}(u,v))\ |\mathbf{r}_u\times\mathbf{r}_v|dA$
- Flux surface integral: $\iint_S \mathbf{F}\cdot\mathbf{n}\ d\sigma = \iint_S \mathbf{F}\cdot d\boldsymbol{\sigma} = \iint_R \mathbf{F}(\mathbf{r}(u,v))\cdot(\mathbf{r}_u\times\mathbf{r}_v)\ dA$
- Stokes' Theorem: If S is a piecewise smooth oriented surface bounded by a piecewise smooth curve C and $\mathbf{F}$ is a vector field whose components have continuous partial derivatives on an open region containing S, then

$$\int_C \mathbf{F}\cdot\mathbf{T}\ ds = \iint_S (\nabla\times\mathbf{F})\cdot\mathbf{n}\ d\sigma.$$

- Divergence Theorem: If S is a piecewise smooth closed oriented surface enclosing a volume D and $\mathbf{F}$ is a vector field whose components have continuous partial derivatives on D, then

$$\iint_S \mathbf{F}\cdot\mathbf{n}\ d\sigma = \iiint_D \nabla\cdot\mathbf{F}\ dV.$$

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