MATH 2550 ~~G~~/J w/ Dr. Sal Barone

- Dr. Barone, Prof. Sal, or just Sal, as you prefer

Daily Announcements & Reminders:

MATH 2551 J/HP Spring 2026 Calendar					
	Date	Lecture/Studio Topics	Assignments	Homework Due	Other Notes
Week 1	Mon Jan 12	Studio: Review prerequisites			PA#1
	Tue Jan 13	Lecture: 12.1			
	Wed Jan 14	Studio: 12.1			
	Thu Jan 15	Lecture: 12.4, 12.5			

Goals for ~~Today:~~

Sections 12.1, 12.4, 12.5

- Set classroom norms
- Describe the big-picture goals of the class
- Review $\mathbb{R}^3$ and the dot product
- Introduce the cross product and its properties

Class Values/Norms:

I will also make mistakes.

- Mistakes are a learning opportunity
- Mathematics is collaborative
- Make sure everyone is included
- Criticize ideas, not people
- Be respectful of everyone
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Course Syllabus

Syllabus from Sal's website: Course Syllabus
https://sbarone7.math.gatech.edu/ma2551s26_syllabus.pdf

Access all course documents on Sal's website: MA 2551 Multivariable Calculus - Spring 2026 J/HP
https://sbarone7.math.gatech.edu/ma2551s26.html

Course Summary:

Date	Details	Due
Mon Jan 12, 2026	PA#1A (T)	due by 11:59pm
Wed Jan 21, 2026	PA#2A (T)	due by 11:59pm
Fri Jan 30, 2026	Quiz 1	due by 11:59pm
Thu Feb 5, 2026	Exam 1	due by 2pm
	WeBWorK Set #1 deadline date	due by 11:59pm
Wed Feb 11, 2026	PA#3A (T)	due by 11:59pm
Fri Feb 20, 2026	Quiz 2	due by 11:59pm
Wed Feb 25, 2026	PA#4A (T)	due by 11:59pm
Fri Mar 6, 2026	Quiz 3	due by 11:59pm
Thu Mar 12, 2026	Exam 2	due by 2pm
	WeBWorK Set #2 deadline date	due by 11:59pm
Wed Mar 18, 2026	PA#5A (T)	due by 11:59pm
Wed Apr 1, 2026	PA#6A (T)	
Fri Apr 10, 2026	Quiz 4	
Wed Apr 15, 2026	PA#7A (T)	

Course assignments are not weighted.

Multivariable Calculus J/HP

Welcome to Math 2551 in Spring of 2026! Please read the syllabus carefully and let me know ASAP if you have any questions or concerns. It is your responsibility to be aware of all course policies.

Lecture Streaming and Recording Links

Lecture J/HP streamed on Zoom on TTh 2pm-3:15pm
https://gatech.zoom.us/j/97411643686

Recordings posted on my MediaSpace channel
https://mediaspace.gatech.edu/channel/channelid/297898903

Office hours

Office hours are available M-Th 9am-7pm and F 9am-5pm with UTA Office Hours in Skiles 230 (walk-ins allowed, or for individual or small group bookings use this link), and also in the Math Lab M-Th 12:30-6pm and F 12:30-3:30pm in Clough 280 (walk-ins only). For meeting with a specific TA or instructor please see below.

TA/Instructor	Section(s)	Email	Office Hours
Sal Barone	J/HP	sbarone@math.gatech.edu	T 11:00-1:00 Skiles 013
Anand Hande	J01/J02	ahande3@gatech.edu	TBD
Shenduo Zhang	J03/J04	szhang705@gatech.edu	TBD
Seth Brunner	J05/HP1	sbrunner7@gatech.edu	TBD

Course Syllabus

https://sbarone7.math.gatech.edu/ma2551s26_syllabus.pdf

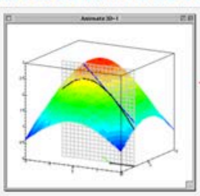
Sal's website

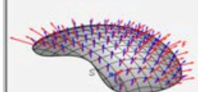
The course syllabus and many other important course documents can all be found on Sal's website:

https://sbarone7.math.gatech.edu/ma2551s26.html

Georgia Tech College of Sciences
School of Mathematics

MA 2551 Multivariable Calculus - Spring 2026 J/HP





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Lecture meeting times: Lecture J/HP on TTh 2:00-2:50pm in CoC 16
Instructor: Sal Barone
Office: Skiles 013 and https://gatech.zoom.us/my/sbarone7
Office hours: T 11:00am-1:00pm (and by appointment, email me) - all office hours are in-person.
email: sbarone at math.gatech.edu
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Links

- See Canvas Welcome page
- Dr. H's Website for Math 2551 resources
- Dr. H's Pre-lecture recordings
- Dr. H's Lecture recordings from Fall 2024
- Sal's Zoom set up a time via email/Discord message to schedule or to "knock on my door"
- Annotated GoodNotes Lecture Slides
- green and blue and red for phone screen responses
- mrgencyXit Discord chat room

Course Documents

- Course Syllabus for Spring 2026 J/HP
- Course Calendar for Spring 2026 J/HP
- Piazza! discussion board for questions outside of class time
- MyMathLab instructions for online textbook access

single variable calculus

Big Idea: Extend differential & integral calculus.

What are some key ideas from these two courses?

volumes?

<u>Differential Calculus</u> 1551

power rule ✓

linear approx ✓

Riemann sums ×

optimization ?

<u>Integral Calculus</u>

Taylor series ×

Series in general ×

U-sub ✓

IBP ?

partial frac ×

trig sub ×

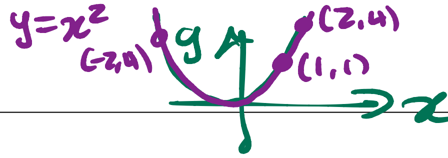
polar coords ✓

Before: we studied **single-variable functions** $f : \mathbb{R} \to \mathbb{R}$ like $f(x) = 2x^2 - 6$.

Now: we will study **multi-variable functions** $f : \mathbb{R}^n \to \mathbb{R}^m$: each of these functions is a rule that assigns one output vector with m entries to each input vector with n entries.

e.g. $f(x,y) = (x^2 - y,\ y^2 + x,\ xy - 1)$

$f : \mathbb{R}^2 \to \mathbb{R}^3$ multivariable function w/ 2 inputs & 3 outputs.

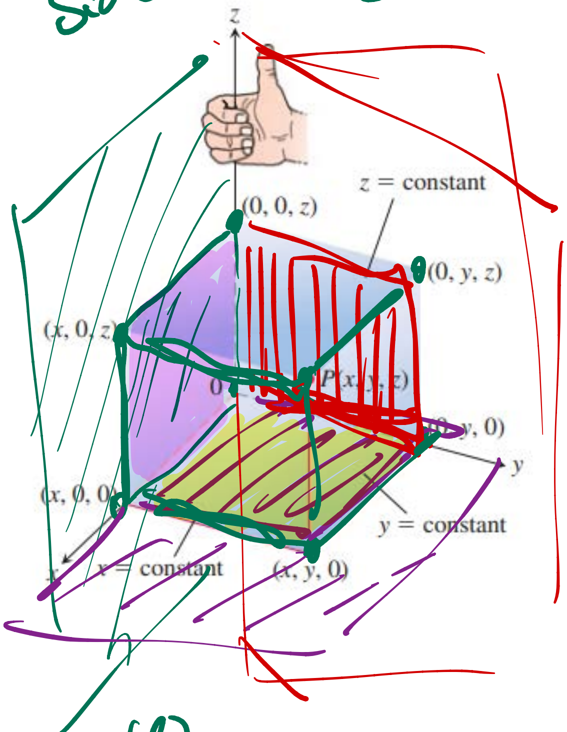


$f(x)=x^2$

§12.1: Three-Dimensional Coordinate Systems

Side wall $y=0$.

Floor $z=0$

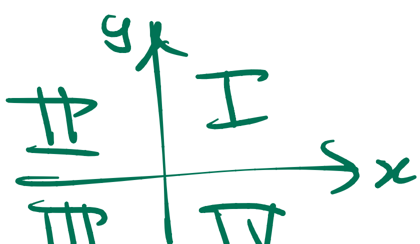


back wall $x=0$

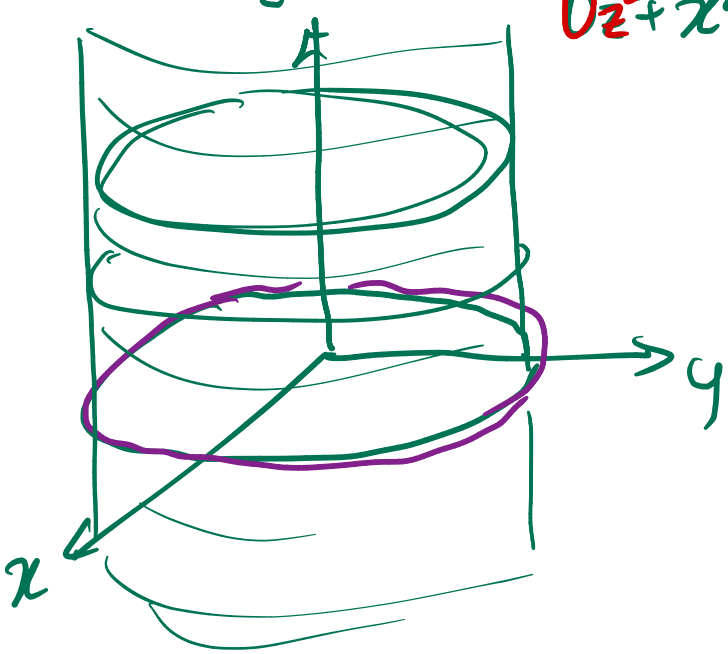
3 coordinate planes defined by setting one of the vars to zero.

(8) Octants vs. (4) quadrant

$\boxed{x \geq 0, y \geq 0, z \geq 0}$ First octant.



Question: What shape is the set of solutions $(x, y, z) \in \mathbb{R}^3$ to the equation $x^2+y^2 = 1$?



$0z^2 + x^2+y^2=1$ only look at $z=0$ solns then you get a circle of radius 1 in the floor

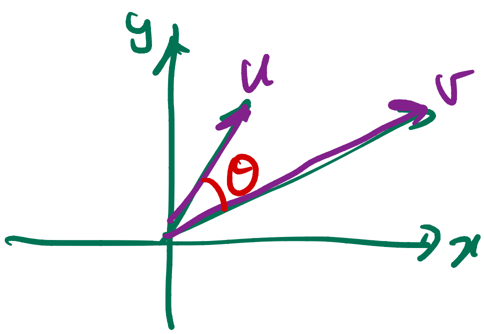
§12.3, 12.4: Dot & Cross Products

Definition 1. The **dot product** of two vectors $\mathbf{u} = \langle u_1, u_2, \dots, u_n\rangle$ and $\mathbf{v} = \langle v_1, v_2, \dots, v_n\rangle$ is

u_1 $\vec{u}$

$$\mathbf{u}\cdot\mathbf{v} = u_1v_1 + u_2v_2 + \cdots + u_nv_n$$

This product tells us about orthogonal / angle between orthogonal



$$\vec{u}\cdot\vec{v} = \|\vec{u}\| * \|\vec{v}\| * \cos\theta$$

$$\vec{u}\cdot\vec{u} = u_1^2 + u_2^2 + \cdots + u_n^2$$
$$= \|\vec{u}\|^2$$

In particular, two vectors are **orthogonal** if and only if their dot product is 0.

Example 2. Are $\mathbf{u} = \langle 1, 1, 4\rangle$ and $\mathbf{v} = \langle -3, -1, 1\rangle$ orthogonal?

\langle
\rangle
"angle brackets"

Compute $\vec{u}\cdot\vec{v} \stackrel{?}{=} 0$

$$\langle 1, 1, 4\rangle \cdot \langle -3, -1, 1\rangle = (1)(-3) + (1)(-1) + (4)(1)$$
$$= -3 - 1 + 4 = 0$$

So $\vec{u}$ is orthogonal to $\vec{v}$

don't write:

$(1,1,4)\cdot(-3,-1,1) = \cdots$ since these are POINTS.

parentheses not ok for vectors.

(0.) $\vec{u} \times \vec{v}$ needs to be orthogonal to both $\vec{u}$ & $\vec{v}$

Goal: Given two vectors, produce a vector orthogonal to both of them in a "nice" way.

We want our "product" to play nice w/ vector addition & scalar multiplication.

1. $\vec{u} \times (\vec{v} + \vec{w}) = \vec{u} \times \vec{v} + \vec{u} \times \vec{w}$

2. $(c\vec{u}) \times \vec{v} = c(\vec{u} \times \vec{v})$

Definition 3. The **cross product** of two vectors $\mathbf{u} = \langle u_1, u_2, u_3 \rangle$ and $\mathbf{v} = \langle v_1, v_2, v_3 \rangle$ in $\mathbb{R}^3$ is

$$\mathbf{u} \times \mathbf{v} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ u_1 & u_2 & u_3 \\ v_1 & v_2 & v_3 \end{vmatrix}$$

take the determinant

$\hat{i}$ i-hat $\hat{i} = \langle 1, 0, 0 \rangle$

$\hat{j}$ j-hat $\hat{j} = \langle 0, 1, 0 \rangle$

$\hat{k}$ k-hat $\hat{k} = \langle 0, 0, 1 \rangle$

Example 4. Find $\langle 1, 2, 0\rangle \times \langle 3, -1, 0\rangle$.

$\vec{u} = \langle 1,2,0\rangle$

$\vec{v} = \langle 3,-1,0\rangle$

So $\vec{u}\times\vec{v} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 2 & 0 \\ 3 & -1 & 0 \end{vmatrix}$

$$= \hat{i}\begin{vmatrix} 2 & 0 \\ -1 & 0 \end{vmatrix} - \hat{j}\begin{vmatrix} 1 & 0 \\ 3 & 0 \end{vmatrix} + \hat{k}\begin{vmatrix} 1 & 2 \\ 3 & -1 \end{vmatrix}$$

$$= \hat{i}\left(2*0 - 0(-1)\right) - \hat{j}\left(1*0 - 0*3\right) + \hat{k}\left(-1-6\right)$$

$$= 0\hat{i} - 0\hat{j} - 7\hat{k}$$

$$= -7\hat{k} = -7\langle 0,0,1\rangle = \boxed{\langle 0,0,-7\rangle}$$

Example 5. *You try it!* Find $\langle 2,1,0\rangle \times \langle 1,2,1\rangle$.

$$\begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ 2 & 1 & 0 \\ 1 & 2 & 1 \end{vmatrix} = \hat{\imath}(1-0) - \hat{\jmath}(2-0) + \hat{k}(4-1)$$

$$= \hat{\imath} - 2\hat{\jmath} + 3\hat{k}$$

$$= \boxed{\langle 1, -2, 3\rangle}$$

Some common [AJN] things to look out for.

[A] Accuracy

- simplify answer
- box answer ✓

$\cos(\pi/2)$ ✗ $1/\sqrt{2}$ ✓

$\sqrt{4 * 3}$ ✗

[J] Justification

- minus sign on **j** component
- show intermediate steps

[N] Notation

- use = sign for expressions that are equal
- vector notation vs. point notation

A Geometric Interpretation of $\mathbf{u} \times \mathbf{v}$

The cross product $\mathbf{u} \times \mathbf{v}$ is the vector

$$\mathbf{u} \times \mathbf{v} = (|\mathbf{u}||\mathbf{v}| \sin \theta)\mathbf{n}$$

where $\mathbf{n}$ is a unit vector which is normal to the plane spanned by $\mathbf{u}$ and $\mathbf{v}$.

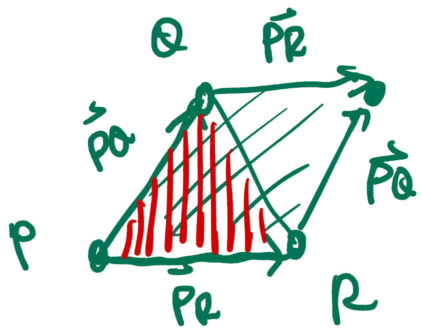
Since $\mathbf{n}$ is a unit vector, the magnitude of $\mathbf{u} \times \mathbf{v}$ is the area of the parallelogram spanned by $\mathbf{u}$ and $\mathbf{v}$.

$$|\mathbf{u} \times \mathbf{v}| = |\mathbf{u}||\mathbf{v}| \sin \theta$$

divide by 2 to get triangle.

Example 5. Find the area of the parallelogram determined by the points P, Q, and R.

$$P(1,1,1),\ Q(2,1,3),\ R(3,-1,1)$$



$\vec{u} = \vec{PQ} = \vec{OQ} - \vec{OP}$

$= \langle 2-1, 1-1, 3-1 \rangle$

$= \langle 1, 0, 2 \rangle$

$\vec{v} = \vec{PR} = \langle 3-1, -1-1, 1-1 \rangle$

$= \langle 2, -2, 0 \rangle$

So $\vec{PQ} \times \vec{PR} = \langle 1,0,2 \rangle \times \langle 2,-2,0 \rangle$

$$= \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 0 & 2 \\ 2 & -2 & 0 \end{vmatrix} = \hat{i}(0-(-4)) - \hat{j}(0-4) + \hat{k}(-2-0)$$

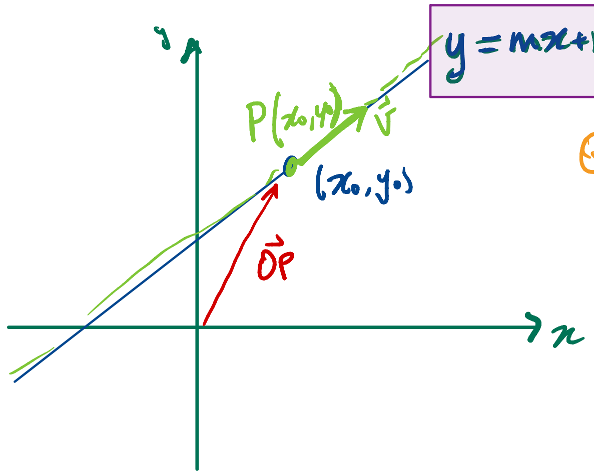
$= \langle 4, 4, -2 \rangle$

So Area $= \|\langle 4,4,-2 \rangle\| = \sqrt{4^2+4^2+(-2)^2} = \sqrt{16+16+4} = \sqrt{36} = \boxed{6}$

§12.5 Lines & Planes

P any point on the line

Lines in $\mathbb{R}^2$, a new perspective:



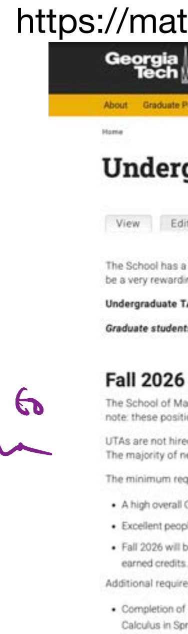
$\boxed{y = mx+b}$ vs. $\boxed{\ell(t) = \vec{OP} + t\vec{v} \quad t\in\mathbb{R}}$

Q: what's this mean geometrically?

vector equation defining the line.

don't forget $t\in\mathbb{R}$.

any vector parallel to the line



Example 7. Find a vector equation for the line that goes through the points $P = (1, 0, 2)$ and $Q = (-2, 1, 1)$.

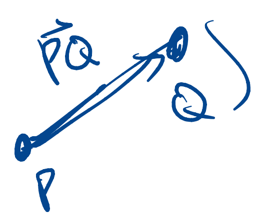
$\ell(t) = \vec{OP} + t\vec{v}$, $t\in\mathbb{R}$ (answer type)

$\vec{OP} = \langle 1,0,2\rangle$ $\quad \vec{v} = \vec{PQ} = \langle -2,1,1\rangle - \langle 1,0,2\rangle$

$= \langle -3, 1, -1\rangle$

$$\boxed{\begin{aligned}\ell(t) &= \langle 1,0,2\rangle + t\langle -3,1,-1\rangle \\ t&\in\mathbb{R}^1\end{aligned}}$$

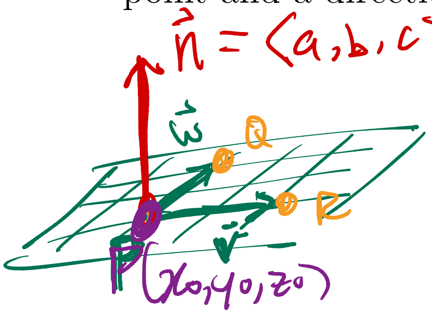
(want just



$t\in[0,1]$ instead)

Planes in $\mathbb{R}^3$

Conceptually: A plane is determined by either three points in $\mathbb{R}^3$ or by a single point and a direction $\mathbf{n}$, called the *normal vector*.



Want to define a plane in $\mathbb{R}^3$

Some options:

(0) * Plane equation

(1) * point P & linearly ind vectors $\vec{v}, \vec{w}$ which are parallel to plane

(2) * 3 points in the plane

(3) * point P & normal vector $\vec{n}$

Algebraically: A plane in $\mathbb{R}^3$ has a *linear* equation (back to Linear Algebra! imposing a single restriction on a 3D space leaves a 2D linear space, i.e. a plane)

(1) $\boxed{ax + by + cz = d}$ plane equation in 3 vars x, y, z.

idea: a point (x_0, y_0, z_0) is on the plane if it satisfies the plane equation.

Notice that $\langle x_0, y_0, z_0 \rangle$ corresponds to a point on the plane if

$$\langle a, b, c \rangle \cdot \langle x_0, y_0, z_0 \rangle = d \quad (1)$$

Some fixed point on the plane

Since $\iff ax_0 + by_0 + cz_0 = d$

So $\langle a, b, c \rangle \cdot \langle x, y, z \rangle = d \quad (2)$

Some other arbitrary point.

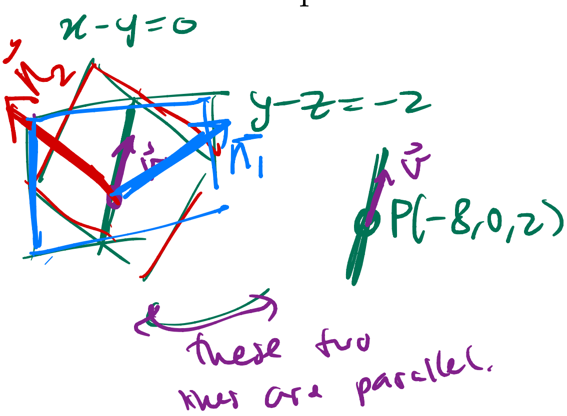
Combining (1) & (2)

$$\langle a, b, c \rangle \cdot \langle x, y, z \rangle - \langle a, b, c \rangle \cdot \langle x_0, y_0, z_0 \rangle = d - d$$

$$\iff \boxed{\langle a, b, c \rangle \cdot \langle x - x_0, y - y_0, z - z_0 \rangle = 0}$$

(2) $$\iff \boxed{a(x - x_0) + b(y - y_0) + c(z - z_0) = 0}$$

Example 8. Consider the planes $y - z = -2$ and $x - y = 0$. Show that the planes intersect and find an equation for the line passing through the point $P = (-8, 0, 2)$ which is parallel to the line of intersection of the planes.



META (1) Find the vector $\vec{v}$ which is parallel to the line of intersection of the planes

(2) use $\ell(t) = \vec{OP} + t\vec{v}$, $t \in \mathbb{R}$ w/ $P(-8,0,2)$ & $\vec{v}$ from (1).

key ideas:
$\vec{v}$ is parallel to $\vec{n}_1 \times \vec{n}_2$

plane 1: $0x + 1y - 1z = -2$ $\quad \vec{n}_1 = \langle 0, 1, -1 \rangle$

plane 2: $1x - 1y + 0z = 0$ $\quad \vec{n}_2 = \langle 1, -1, 0 \rangle$

① planes must intersect b/c $\vec{n}_1 \neq c\,\vec{n}_2$.

② compute

$$\vec{n}_1 \times \vec{n}_2 = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 0 & 1 & -1 \\ 1 & -1 & 0 \end{vmatrix} = \hat{i}(0-1) - \hat{j}(0+1) + \hat{k}(0-1)$$
$$= \langle -1, -1, -1 \rangle$$

So line of intersection of two planes is parallel to $\vec{v} = \langle -1, -1, -1 \rangle$

③ line parallel to $\vec{v}$ and through $P(-8,0,2)$ is

$$\boxed{\ell(t) = \langle -8, 0, 2 \rangle + t\langle -1, -1, -1 \rangle,\ t \in \mathbb{R}}$$

Example 9. *You try it!* Find the plane containing the lines parameterized by

$$\begin{aligned} \ell_1(t) &= \langle 1,1,1\rangle + t\langle 2,1,0\rangle, \qquad -\infty < t < \infty \\ \ell_2(s) &= \langle 0,-1,0\rangle + s\langle 1,2,1\rangle, \qquad -\infty < s < \infty \end{aligned}$$

Give your answer in the form $Ax+By+Cz=D$ or $a(x-x_0)+b(y-y_0)+c(z-z_0)=0$.

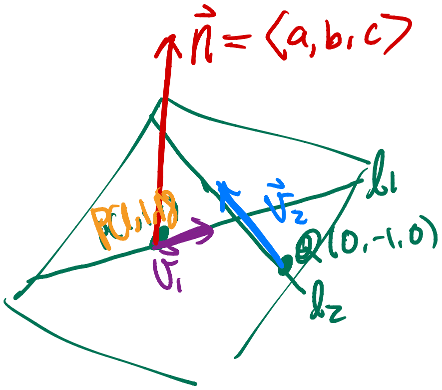
Example 9. *You try it!* Find the plane containing the lines parameterized by

$$\ell_1(t) = \langle 1,1,1\rangle + t\langle 2,1,0\rangle, \qquad -\infty < t < \infty$$
$$\ell_2(s) = \langle 0,-1,0\rangle + s\langle 1,2,1\rangle, \qquad -\infty < s < \infty$$

$\vec{v}_1 = \langle 2,1,0\rangle$

$\vec{v}_2 = \langle 1,2,1\rangle$

Give your answer in the form $Ax+By+Cz = D$ or $a(x-x_0)+b(y-y_0)+c(z-z_0) = 0$.



META (1) Find $\vec{n} = \langle a,b,c\rangle$ normal to the plane

(i.e., orthogonal to $\vec{v}_1$ & $\vec{v}_2$)

(2) use plane equation

$$a(x-x_0)+b(y-y_0)+c(z-z_0)=0$$

So $\vec{n} = \vec{v}_1 \times \vec{v}_2 = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 1 & 0 \\ 1 & 2 & 1 \end{vmatrix} = \hat{i}(1-0) - \hat{j}(2-0) + \hat{k}(4-1)$

$= \langle 1,-2,3\rangle$

So plane equation using formula $a(x-x_0)+b(y-y_0)+c(z-z_0)=0$ w/ $\vec{n} = \langle 1,-2,3\rangle$ & $P(1,1,1)$ is

$$\boxed{(x-1)-2(y-1)+3(z-1)=0}$$

OR $x-1-2y+2+3z-3=0$

$\Rightarrow x-2y+3z = 1-2+3$

$$\Rightarrow \boxed{x-2y+3z=2}$$

either way ok