MATH 2550 G/J w/ Dr. Sal Barone

- Dr. Barone, Prof. Sal, or just Sal, as you prefer

Daily Announcements & Reminders:

Goals for Today: Sections 12.1, 12.4, 12.5

- Set classroom norms
- Describe the big-picture goals of the class
- Review $\mathbb{R}^3$ and the dot product
- Introduce the cross product and its properties

Class Values/Norms:

- Mistakes are a learning opportunity
- Mathematics is collaborative
- Make sure everyone is included
- Criticize ideas, not people
- Be respectful of everyone
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Big Idea: Extend differential & integral calculus.

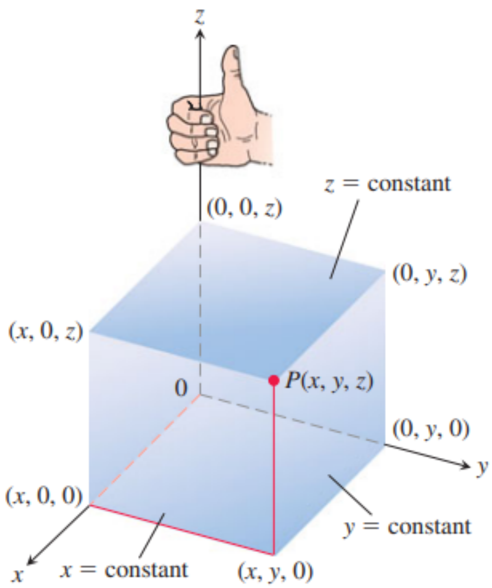
What are some key ideas from these two courses?

Differential Calculus	Integral Calculus

Before: we studied **single-variable functions** $f : \mathbb{R} \to \mathbb{R}$ like $f(x) = 2x^2 - 6$.

Now: we will study **multi-variable functions** $f : \mathbb{R}^n \to \mathbb{R}^m$: each of these functions is a rule that assigns one output vector with m entries to each input vector with n entries.

§12.1: Three-Dimensional Coordinate Systems



Question: What shape is the set of solutions $(x, y, z) \in \mathbb{R}^3$ to the equation $x^2+y^2 = 1$?

§12.3, 12.4: Dot & Cross Products

Definition 1. The **dot product** of two vectors $\mathbf{u} = \langle u_1, u_2, \ldots, u_n \rangle$ and $\mathbf{v} = \langle v_1, v_2, \ldots, v_n \rangle$ is

$$\mathbf{u} \cdot \mathbf{v} = \underline{\hspace{8cm}}$$

This product tells us about ____.

In particular, two vectors are **orthogonal** if and only if their dot product is ______.

Example 2. Are $\mathbf{u} = \langle 1, 1, 4 \rangle$ and $\mathbf{v} = \langle -3, -1, 1 \rangle$ orthogonal?

Goal: Given two vectors, produce a vector orthogonal to both of them in a "nice" way.

1.

2.

Definition 3. The **cross product** of two vectors $\mathbf{u} = \langle u_1, u_2, u_3 \rangle$ and $\mathbf{v} = \langle v_1, v_2, v_3 \rangle$ in $\mathbb{R}^3$ is

$$\mathbf{u} \times \mathbf{v} = \underline{\qquad\qquad\qquad\qquad\qquad\qquad}$$

Example 4. Find $\langle 1, 2, 0 \rangle \times \langle 3, -1, 0 \rangle$.

Example 5. *You try it!* Find $\langle 2, 1, 0\rangle \times \langle 1, 2, 1\rangle$.

Some common [AJN] things to look out for.

[A] Accuracy

- simplify answer
- box answer

[J] Justification

- minus sign on $\mathbf{j}$ component
- show intermediate steps

[N] Notation

- use = sign for expressions that are equal
- vector notation vs. point notation

A Geometric Interpretation of $\mathbf{u} \times \mathbf{v}$

The cross product $\mathbf{u} \times \mathbf{v}$ is the vector

$$\mathbf{u} \times \mathbf{v} = (|\mathbf{u}||\mathbf{v}| \sin\theta)\mathbf{n}$$

where $\mathbf{n}$ is a unit vector which is normal to the plane spanned by $\mathbf{u}$ and $\mathbf{v}$.

Since $\mathbf{n}$ is a unit vector, the magnitude of $\mathbf{u} \times \mathbf{v}$ is the area of the parallelogram spanned by $\mathbf{u}$ and $\mathbf{v}$.

$$|\mathbf{u} \times \mathbf{v}| = |\mathbf{u}||\mathbf{v}| \sin\theta$$

Example 5. Find the area of the parallelogram determined by the points P, Q, and R.

$$P(1, 1, 1),\ Q(2, 1, 3),\ R(3, -1, 1)$$

§12.5 Lines & Planes

Lines in $\mathbb{R}^2$, a new perspective:

Example 7. Find a vector equation for the line that goes through the points $P = (1, 0, 2)$ and $Q = (-2, 1, 1)$.

Planes in $\mathbb{R}^3$

Conceptually: A plane is determined by either three points in $\mathbb{R}^3$ or by a single point and a direction $\mathbf{n}$, called the *normal vector*.

Algebraically: A plane in $\mathbb{R}^3$ has a *linear* equation (back to Linear Algebra! imposing a single restriction on a 3D space leaves a 2D linear space, i.e. a plane)

Example 8. Consider the planes $y - z = -2$ and $x - y = 0$. Show that the planes intersect and find an equation for the line passing through the point $P = (-8, 0, 2)$ which is parallel to the line of intersection of the planes.

Example 9. *You try it!* Find the plane containing the lines parameterized by

$$\begin{aligned} \ell_1(t) &= \langle 1,1,1\rangle + t\langle 2,1,0\rangle, \qquad -\infty < t < \infty \\ \ell_2(s) &= \langle 0,-1,0\rangle + s\langle 1,2,1\rangle, \qquad -\infty < s < \infty \end{aligned}$$

Give your answer in the form $Ax+By+Cz = D$ or $a(x-x_0)+b(y-y_0)+c(z-z_0) = 0$.