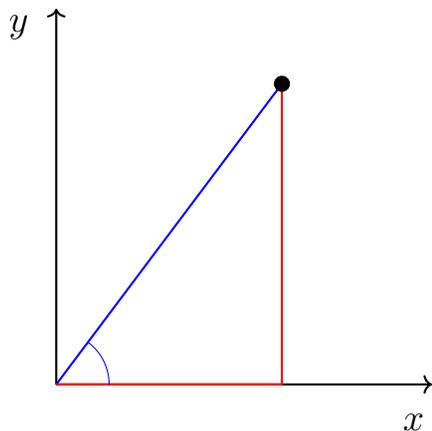


§15.4 Double Integrals in Polar Coordinates

Review of Polar Coordinates



Cartesian coordinates: Give the distances in _____ and _____ directions from _____

Polar coordinates:

- r = distance from _____ to _____
- θ = angle between the ray _____ and the positive _____

We can use trigonometry to go back and forth.

Polar to Cartesian:

$$x = r \cos(\theta) \quad y = r \sin(\theta)$$

Cartesian to Polar:

$$r^2 = x^2 + y^2 \quad \tan(\theta) = \frac{y}{x}$$

Example 94. a) Find a set of polar coordinates for the point $(x, y) = (1, 1)$.

b) Graph the set of points (x, y) that satisfy the equation $r = 2$ and the set of points that satisfy the equation $\theta = \pi/4$ **in the xy -plane.**

c) Write the function $f(x, y) = \sqrt{x^2 + y^2}$ in polar coordinates.

d) *You try it!* Write a Cartesian equation describing the points that satisfy $r = 2 \sin(\theta)$.

Goal: Given a region R in the xy -plane described in polar coordinates and a function $f(r, \theta)$ on R , compute $\iint_R f(r, \theta) dA$.

Example 95. Compute the area of the disk of radius 5 centered at $(0, 0)$.

Remember: In polar coordinates, the area form $dA =$ _____

Goal: Given a region R in the xy -plane described in polar coordinates and a function $f(r, \theta)$ on R , compute $\iint_R f(r, \theta) dA$.

Example 96. Compute the area of the disk of radius 5 centered at $(0, 0)$.

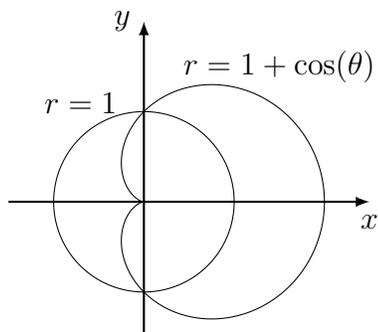
Cont.

Remember: In polar coordinates, the area form $dA =$ _____

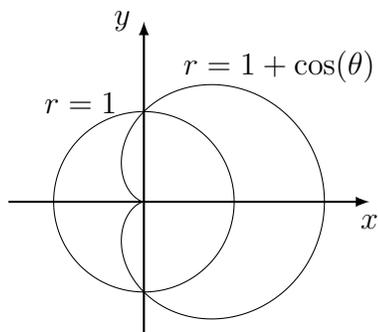
Example 96. Compute $\iint_D e^{-(x^2+y^2)} dA$ on the washer-shaped region $1 \leq x^2 + y^2 \leq 4$.

Example 97. Compute the area of the smaller region bounded by the circle $x^2 + (y - 1)^2 = 1$ and the line $y = x$.

Example 98. *You try it!* Write an integral for the volume under $z = x$ on the region between the cardioid $r = 1 + \cos(\theta)$ and the circle $r = 1$, where $x \geq 0$.



Example 98. *You try it!* Write an integral for the volume under $z = x$ on the region between the cardioid $r = 1 + \cos(\theta)$ and the circle $r = 1$, where $x \geq 0$.



Example 100. Convert the integral in polar coordinates to an equivalent integral in cartesian coordinates, but do not evaluate. Then, evaluate the original integral to find the value of $\iint_R f(x, y) dA$.

$$\int_{\pi/6}^{\pi/2} \int_1^{\csc \theta} r^2 \cos \theta dr d\theta$$

Tips and tricks

For horizontal lines such as $x = 2$:

For vertical lines such as $y = 1$ (e.g., Example [100](#)):

For off-set circles such as $x^2 + (y - 1)^2 = 1$ (e.g., Example [98](#)):

Example 101. *You try it!* Find the area of the region R which is the smaller part bounded between the circle $x^2 + y^2 = 4$ and the line $x = 1$.

Example 101. *You try it!* Find the area of the region R which is the smaller part bounded between the circle $x^2 + y^2 = 4$ and the line $x = 1$.

Math 2551 Worksheet: Exam 2 Review

- Which of the following statements are true if $f(x, y)$ is differentiable at (x_0, y_0) ? Give reasons for your answers.
 - If \mathbf{u} is a unit vector, the derivative of f at (x_0, y_0) in the direction of \mathbf{u} is $(f_x(x_0, y_0)\mathbf{i} + f_y(x_0, y_0)\mathbf{j}) \cdot \mathbf{u}$.
 - The derivative of f at (x_0, y_0) in the direction of \mathbf{u} is a vector.
 - The directional derivative of f at (x_0, y_0) has its greatest value in the direction of ∇f .
 - At (x_0, y_0) , the vector ∇f is normal to the curve $f(x, y) = f(x_0, y_0)$.
- Find dw/dt at $t = 0$ if $w = \sin(xy + \pi)$, $x = e^t$, and $y = \ln(t + 1)$.
- Find the extreme values of $f(x, y) = x^3 + y^2$ on the circle $x^2 + y^2 = 1$.
- Test the function $f(x, y) = x^3 + y^3 + 3x^2 - 3y^2$ for local maxima and minima and saddle points and find the function's value at these points.
- Find the points on the surface $xy + yz + zx - x - z^2 = 0$ where the tangent plane is parallel to the xy -plane.
- Evaluate the integral $\int_0^1 \int_{2y}^2 4 \cos(x^2) dx dy$. Describe why you made any choices you did in the course of evaluating this integral.
- If $f(x, y) \geq 2$ for all (x, y) , is it possible that the average value of $f(x, y)$ on a unit disk centered at the origin is $\frac{2}{\pi}$?
- A swimming pool is circular with a 40 foot diameter. The depth is constant along east-west lines and increases linearly from 2 feet at the south end to 7 feet at the north end. Find the volume of water in the pool.