Quiz 3

Be sure to follow the quiz instructions in order to avoid a deduction in points. Submissions are due in Gradescope by 11:59pm on Friday; no late work is accepted.

Name:

Question #1: Find the average height of the paraboloid $z = x^2 + y^2$ over the rectangle $R = [0, 2] \times [0, 4]$. [AJN]

$$\boxed{20/3}$$

$$\text{Avg} = \frac{1}{\text{Area}R} \iint_R f(x,y)\, dA$$

$$\text{Area}R = 2 * 4 = 8$$

$$\iint_R f(x,y)\, dA = \int_0^2 \int_0^4 x^2 + y^2 \, dy\, dx$$

$$= \int_0^2 x^2 y + \frac{1}{3} y^3 \Big|_0^4$$

$$= \int_0^2 \left(4x^2 + \frac{64}{3}\right) - (0+0)\, dx$$

$$= \frac{4}{3}x^3 + \frac{64}{3}x \Big|_0^2$$

$$= \left(\frac{4}{3}(8) + \frac{64}{3}(2)\right) - (0+0)$$

$$= \frac{32}{3} + \frac{128}{3} = 160/3$$

So $\text{Avg} = \frac{1}{8} * \frac{160}{3}$

$$= \boxed{20/3}$$

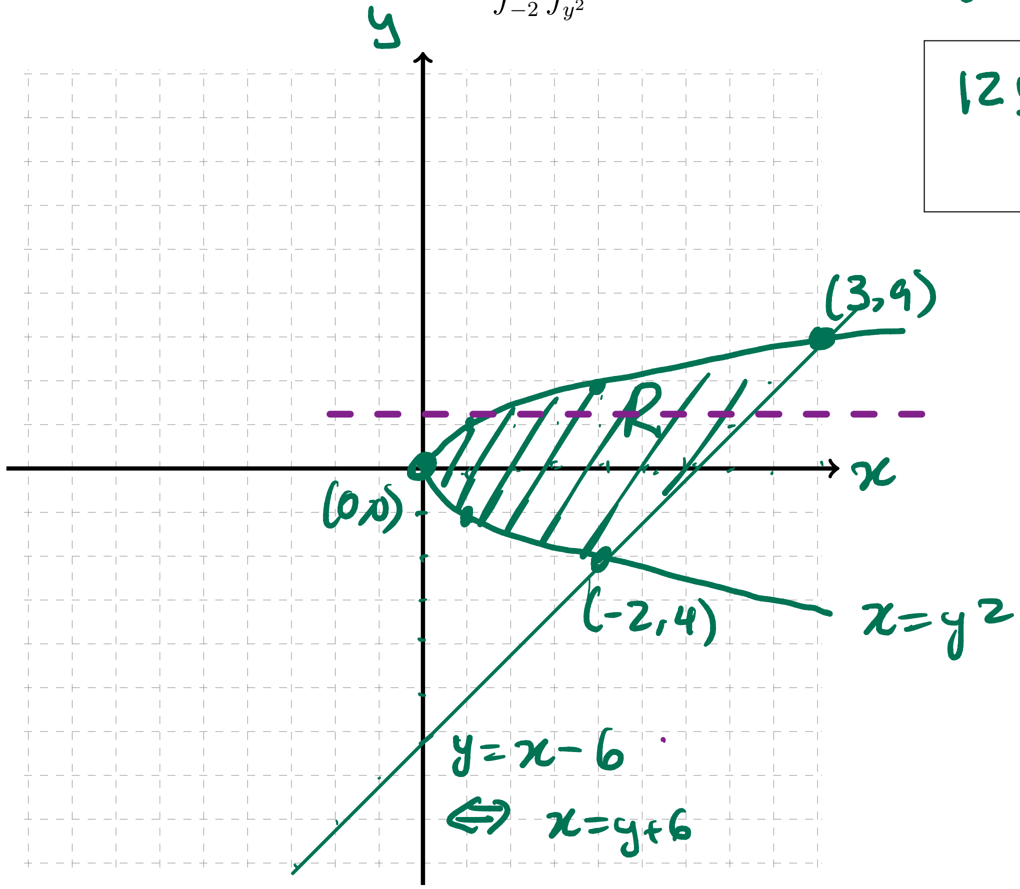
Question #2: The integral below gives the area of a region R in the xy-plane. Sketch the region R, labeling each boundary curve with its equation, as well as providing labels for the axes, as well as the x-intercepts, y-intercepts, and the points of intersection of the boundary curves. Then, find the area of the region R. [AJN]

curves ① $x=y+6 \Rightarrow y=x-6$

$$\int_{-2}^{3}\int_{y^2}^{y+6} dx\, dy$$

② $x=y^2$

$\boxed{125/6}$



y-range $[-2,3]$ and x-range $[y^2, y+6]$

$$\text{Area}(R) = \iint_R 1\, dA = \int_{-2}^{3}\int_{y^2}^{y+6} 1\, dx\, dy$$

$$= \int_{-2}^{3} x \Big|_{y^2}^{y+6} = \int_{-2}^{3} (y+6) - y^2\, dy$$

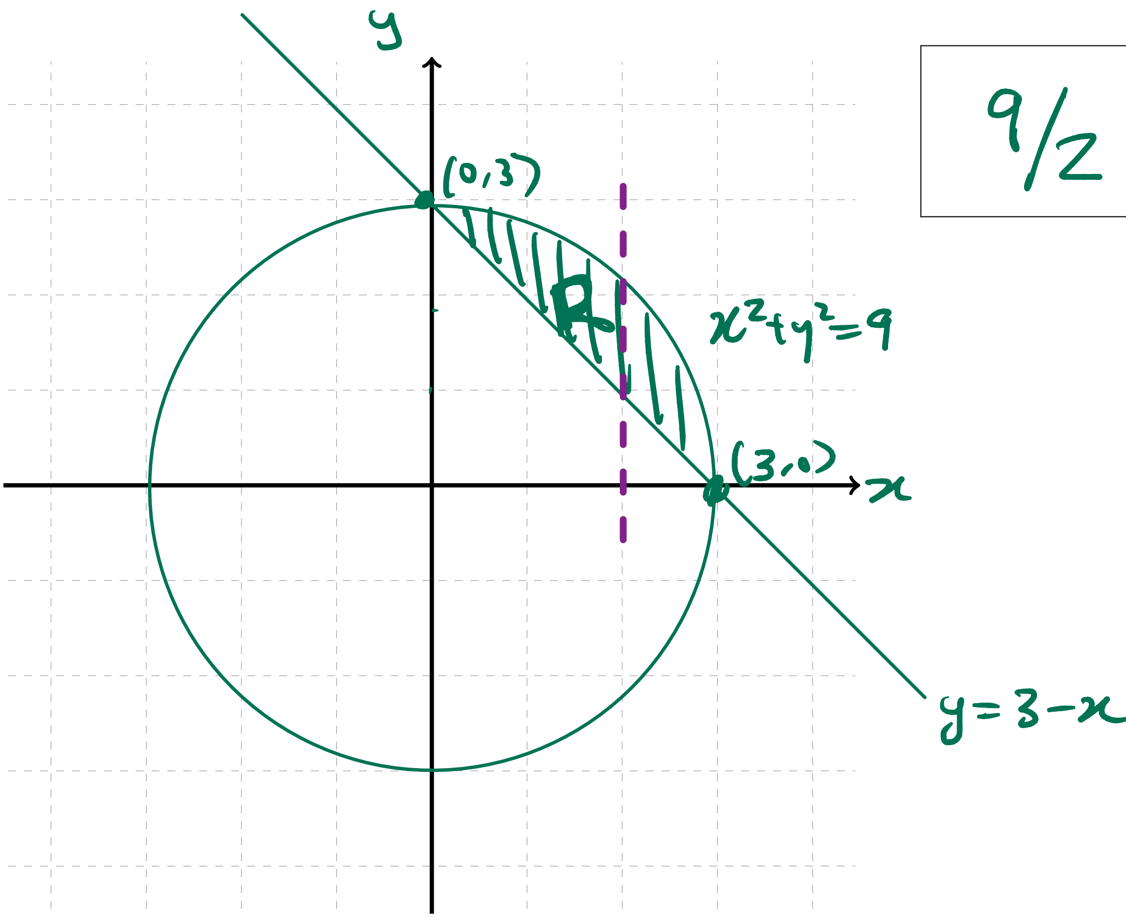
$$= \frac{1}{2}y^2 + 6y - \frac{1}{3}y^3 \Big|_{-2}^{3} = \left(\frac{9}{2} + 18 - \cancel{\frac{27}{3}}^{\,9}\right) - \left(\cancel{\frac{4}{2}}^{\,2} - 12 + \frac{8}{3}\right)$$

$$= \frac{9}{2} + 9 + 10 - 8/3$$

$$= \frac{27 + 114 - 16}{6}$$

$$= \boxed{125/6}$$

Question #3: Let R be the region in the first quadrant that lies inside the circle $x^2+y^2=9$ and above the line $y=3-x$, and suppose $f(x,y)=x$. Sketch the region R on the axes provided, and make sure to label the axes, each boundary curve, and the intercepts of the boundary curves in your sketch. Then, find the value of $\iint_R f(x,y)\,dA$. [AJN]



$\boxed{9/2}$

x ranges from $[0,3]$ then y ranges from $[3-x, \sqrt{9-x^2}]$

$$\iint_R f(x,y)\,dA = \int_0^3 \int_{3-x}^{\sqrt{9-x^2}} x\,dy\,dx = \int_0^3 xy \Big|_{3-x}^{\sqrt{9-x^2}} dx$$

$u=9-x^2$
$du=-2x\,dx$

$$= \int_0^3 x\sqrt{9-x^2} - x(3-x)\,dx = \int_0^3 x\sqrt{9-x^2}\,dx + \int_0^3 x^2-3x\,dx$$

$$= -\frac{1}{2}\cdot\frac{2}{3}(9-x^2)^{3/2}\Big|_0^3 + \frac{1}{3}x^3 - \frac{3}{2}x^2\Big|_0^3 = -\frac{1}{3}(0-27) + \left[\left(9-\frac{27}{2}\right)-(0-0)\right]$$

$$= 9+9-\frac{27}{2} = \boxed{9/2}$$