

Taker Name:

Key

GTID: 90

Section:

Grader #1:

GTID: 90

§14.4: Chain Rule

Find Df and Dr for the functions below. Then, find $D(f(r(t)))$ when $t = e$ using the formula $D(f(r(t))) = Df(r(t))Dr(t)$.

$$f(x, y) = 4e^x \ln y, \mathbf{r}(t) = \langle \ln t, t^2 \rangle$$

$$Df = [f_x \ f_y] \quad \& \quad Dr = \begin{bmatrix} x_t \\ y_t \end{bmatrix}$$

$$\therefore Df = \begin{bmatrix} 4e^x \ln y & \frac{4e^x}{y} \end{bmatrix}$$

$$\text{and } Dr = \left\langle \frac{1}{t}, 2t \right\rangle = \begin{bmatrix} 1/t \\ 2t \end{bmatrix}$$

$$\text{and } @ t=0 \quad r(0) = \langle \ln e, e^2 \rangle \\ = \langle 1, e^2 \rangle$$

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$$\therefore Df(r(e)) = Df(1, \frac{1}{2}e^2) = \begin{bmatrix} 4e^1 \ln(e^2) & \frac{4e^1}{e^2} \end{bmatrix} \\ = \begin{bmatrix} 8e & \frac{4}{e} \end{bmatrix}$$

$$\text{and } Dr(e) = \begin{bmatrix} 1/e \\ 2e \end{bmatrix}$$

$$\ln(e^2) = 2 \ln e \\ = 2 \times 1 = 2$$

$$\therefore Df(r(0)) = (8e \frac{4}{e}) \begin{bmatrix} 1/e \\ 2e \end{bmatrix}$$

$$= 8e \times \frac{1}{e} + \frac{4}{e} \times 2e = 8 + 8 = \boxed{16}$$