Taker Name: Key GTID: 90 Section:

Grader #1: GTID: 90

§14.8: Lagrange Multipliers

Use the method of Lagrange multipliers to find the minimum value of $f(x,y)$ subject to the constraint $g(x,y) = k$. Why is there *no maximum value* of $f(x,y)$ subject to this constraint?

$$f(x,y) = x^2 + y^2, \quad \text{subject to } x^2y = 2.$$

Set up $\nabla f = \langle 2x, 2y \rangle$

& $\nabla g = \langle 2xy, x^2 \rangle$

Solve $\begin{cases} ① \; 2x = \lambda 2xy \\ ② \; 2y = \lambda x^2 \\ ③ \; x^2 y = 2 \end{cases}$ Try ① First

$2x - \lambda 2xy = 0$

$\Rightarrow 2x(1 - \lambda y) = 0$

So Case I: $x = 0$ or Case II: $\lambda y = 1$

In Case I: $x=0$ so ② $y = 0$, but ③ fails ~~X~~

In Case II: $\begin{cases} ① \; 2x = 2x \\ ② \; 2(1/\lambda) = \lambda x^2 \\ ③ \; x^2(1/\lambda) = 2 \end{cases}$

② says $2 = \lambda^2 x^2$

③ says $2 = x^2/\lambda$

So $\lambda^2 x^2 = x^2/\lambda$

$\Rightarrow \lambda^3 = 1 \Rightarrow \lambda = 1$

and $x = \pm\sqrt{2}$,

and original ② then

$2y = (\pm\sqrt{2})^2$

$\Rightarrow y = 1$.

So get

$(\sqrt{2}, 1)$ & $(-\sqrt{2}, 1)$

Evaluate

(x,y)	$f(x,y)$	$d(x,y)$
$(\sqrt{2}, 1)$	$2+1=3$	$\sqrt{3}$
$(-\sqrt{2}, 1)$	3	$\sqrt{3}$

So MIN distance of $\sqrt{3}$ at $(\sqrt{2},1)$ & $(-\sqrt{2},1)$

(no MAX distance since $x^2y=2$ is UNBOUNDED)

A

J

N

G2:

A

J

N

G3:

A

J

N