Taker Name: Key GTID: 90 Section:

Grader #1: GTID: 90

§16.2: Flow and Flux

w/ outward pointing π_0

Find the Flux of $\mathbf{F}$ across the unit circle C oriented counter-clockwise.

$$\mathbf{F} = \langle 2x, -3y \rangle$$

Formula

$$\text{Flux} = \int_C F \cdot n \, ds$$

$r(t) = \langle \cos t, \sin t \rangle \quad t \in [0, 2\pi]$

$r'(t) = \langle -\sin t, \cos t \rangle$

Set $\vec{n} \sim \langle \cos t, \sin t \rangle$, outward pointing ✓

Then

$$\text{Flux} = \int_0^{2\pi} \langle 2\cos t, -3\sin t \rangle \cdot \langle \cos t, \sin t \rangle \, dt$$

$$= \int_0^{2\pi} 2\cos^2 t - 3\sin^2 t \, dt$$

$$= \int_0^{2\pi} 2\cos^2 t - 3(1 - \cos^2 t) \, dt$$

$$= \int_0^{2\pi} 5\cos^2 t - 3 \, dt = \int_0^{2\pi} 5 \cdot \left(\frac{1 + \cos 2t}{2}\right) - 3 \, dt$$

$$= \int_0^{2\pi} \frac{5}{2}\cos 2t - \frac{1}{2} \, dt = \frac{5}{4}\sin(2t) - \frac{1}{2}t \Big|_0^{2\pi}$$

$$= \left(0 - \frac{1}{2}(2\pi)\right) - (0 - 0) = \boxed{-\pi}$$

A	
J	
N	

G2:

A	
J	
N	

G3:

A	
J	
N	