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§16.3: FToLI and Potential Functions

Find a potential function for $\mathbf{F} = \langle P, Q, R\rangle$ and use FToLI to evaluate the line integral over the curve C parametrized by $\mathbf{r}(t)$, $t \in [0,1]$.

$$\int_C \mathbf{F}\cdot T\,ds = \int_C 2\cos y\,dx + \left(\frac{1}{y} - 2x\sin y\right)\,dy + \frac{1}{z}\,dz,$$

where $\mathbf{r}(0) = (0,2,1)$, $\mathbf{r}(1) = (1,\pi/2,2)$.

$$F = \left\langle 2\cos y,\ \frac{1}{y} - 2x\sin y,\ \frac{1}{z}\right\rangle = \langle P, Q, R\rangle$$

$$f_x = P \Rightarrow f = \int 2\cos y\,dx = 2x\cos y + C(y,z)$$

and $f_y = -2x\sin y + C_y(y,z) = Q \Rightarrow C = \ln y + C(z)$

then $f_z = \frac{1}{z} \Rightarrow C(z) = \ln z + C.$

So $f = 2x\cos y + \ln y + \ln z$, $\nabla f = F$ ✓

So $$\int_{(0,2,1)}^{(1,\pi/2,2)} F\cdot dr = f(1,\pi/2,2) - f(0,2,1)$$

$$= \left[2\cancel{\cos\frac{\pi}{2}}^{\,0} + \ln(\pi/2) + \ln 2\right]$$

$$- \left[0\cos 2 + \ln 2 + \cancel{\ln 1}^{\,0}\right]$$

$$= \ln\pi - \ln 2 + \cancel{\ln 2} - \cancel{\ln 2}$$

$$= \boxed{\ln\pi - \ln 2}$$