

Taker Name:

Key

GTID: 90

Section:

Grader #1:

GTID: 90

§16.3: FToLI and Potential Functions

Find a potential function for $\mathbf{F} = \langle P, Q, R \rangle$ and use FToLI to evaluate the line integral over the curve C parametrized by $\mathbf{r}(t)$, $t \in [0, 1]$.

$$\int_C \mathbf{F} \cdot T \, ds = \int_C (2x \ln y - yz) \, dx + \left(\frac{x^2}{y} - xz \right) dy - xy \, dz,$$

$$\text{where } \mathbf{r}(0) = (1, 2, 1), \mathbf{r}(1) = (2, 1, 1).$$

$$\mathbf{F} = \langle 2x \ln y - yz, \frac{x^2}{y} - xz, -xy \rangle = \langle P, Q, R \rangle$$

$$f_x = P \Rightarrow f = \int (2x \ln y - yz) \, dx = x^2 \ln y - xyz + C(y, z)$$

$$\text{and } f_y = Q \Rightarrow \frac{x^2}{y} - xz + C_y(y, z) = \frac{x^2}{y} - xz \Rightarrow C(y, z) = C(z)$$

$$\text{Then } f_z = R \Rightarrow 0 - xy + C'(z) = -xy \Rightarrow C(z) = C.$$

$$\text{So } f = x^2 \ln y - xyz + C \quad \text{Set } C=0$$

$$\text{Then } \int_{(1,2,1)}^{(2,1,1)} \mathbf{F} \cdot d\mathbf{r} = f(1,2,1) - f(2,1,1)$$

$$= [1 \ln 2 - 1 \cdot 2 \cdot 1] - [2 \ln 1 - 2 \cdot 1 \cdot 1]$$

$$= \ln 2 - 2 + 2$$

$$= \boxed{\ln 2}$$

A

J

N

G2:

A

J

N

G3:

A

J

N