MATH 2551 GT-E Midterm 1
VERSION A
Summer 2025
COVERS SECTIONS 12.1-12.6, 13.1-13.4, 14.1-14.2

Full name: ______________________________ **GT ID:**______________

Honor code statement: I will abide strictly by the Georgia Tech honor code at all times. I will not use a calculator. I do not have a phone within reach, and I will not reference any website, application, or other CAS-enabled service. I will not consult with my notes or anyone during this exam. I will not provide aid to anyone else during this exam.

(   ) All of the knowledge presented in this exam is entirely my own. I am initialing to the left to attest to my integrity.

Read all instructions carefully before beginning.

- Print your name and GT ID neatly above.
- You have 75 minutes to take the exam.
- You may not use aids of any kind.
- Please show your work [J] and annotate your work using proper notation [N].
- Good luck!

Question	Points
1	2
2	2
3	2
4	10
5	10
6	12
7	6
8	6
Total:	50

For problems 1-2 choose whether each statement is true or false. If the statement is *always* true, pick true. If the statement is *ever* false, pick false. For all problems on this page please be sure to neatly fill in the bubble corresponding to your answer choice. [A]

1. (2 points) If If $\mathbf{u}$ and $\mathbf{v}$ are vectors in $\mathbb{R}^3$, then $\mathbf{u} \times \mathbf{v} + \mathbf{v} \times \mathbf{u} = \mathbf{0}$.

   ◯ **TRUE** ◯ **FALSE**

2. (2 points) A particle moving at constant speed must have zero acceleration.

   ◯ **TRUE** ◯ **FALSE**

3. (2 points) Which of the following is true about the quadratic surface $x = y^2 + 2z^2$? [A]

   ◯ **A)** It is a sphere, because all of it's cross sections in the $x = k$, $y = k$, and $z = k$ planes are circles.

   ◯ **B)** It is a hyperbolic paraboloid, because its cross-sections are hyperbolas in the $x = k$ planes and parabolas in the $z = k$ and $y = k$ planes.

   ◯ **C)** It is a cone, because its cross-sections are circles in the $x = k$ planes and straight lines in the $x = k$ and $y = k$ planes.

   ◯ **D)** It is an elliptical paraboloid, because its cross-sections are ellipses in the $x = k$ planes and parabolas in the $z = k$ and $y = k$ planes.

   ◯ **E)** It is a plane, because all of its cross sections in the $x = k$, $y = k$, and $z = k$ planes are straight lines.

4. (10 points) Find the plane containing the lines parameterized by

$$\begin{aligned} \ell_1(t) &= \langle 1,1,1\rangle + t\langle 2,1,0\rangle, \qquad -\infty < t < \infty \\ \ell_2(s) &= \langle 0,-1,0\rangle + s\langle 1,2,1\rangle, \qquad -\infty < s < \infty \end{aligned}$$

Give your answer in the form $Ax+By+Cz=D$ or $a(x-x_0)+b(y-y_0)+c(z-z_0)=0$.

[AJN]

5. Mei is looking for her sister Satsuki in the village of Sayama Hills, and her path up a hill is given by $\mathbf{r}(t) = \langle 2, \frac{1}{2}t^2, \frac{1}{3}t^3 \rangle$, for $0 \leq t \leq 2$, where t is measured in minutes and $\mathbf{r}(t)$ in meters. [AJN]

   (a) (2 points) What is Mei's **position** two minutes into her journey?

   (b) (8 points) How far did Mei travel from time $t = 0$ to time $t = 2$?

6. In this problem, you will work with the spiral plane curve

$$\mathbf{r}(t) = (\cos t + t\sin t)\mathbf{i} + (\sin t - t\cos t)\mathbf{j} + 3\mathbf{k}$$

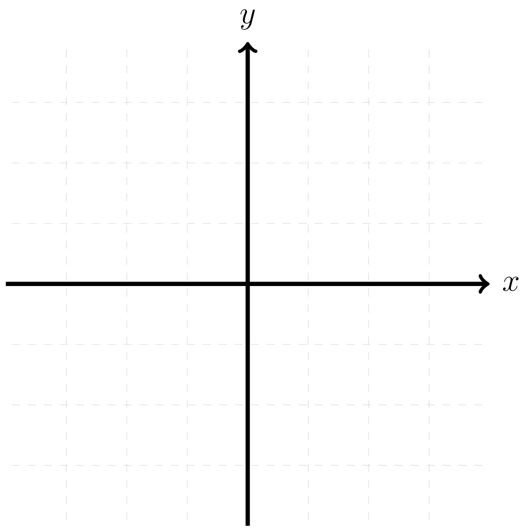
for $t > 0$. *Hint: product rule.* [AJN]

(a) (6 points) Compute the unit tangent vector $\mathbf{T}(t)$. *Hint: product rule.*

(b) (4 points) Compute the principal unit normal vector $\mathbf{N}(t)$.

(c) (2 points) Compute the curvature $\kappa(t)$.

7. (6 points) Let $f(x, y) = \sqrt{4 - x^2 - y^2}$. Graph the domain of f on the provided axes below, and clearly label the axes. Be sure to show all your work in finding the domain. Indicate whether or not each part of the boundary of the domain is included. [AJN]



8. (6 points) Show that the limit does not exist. To receive full credit, you must show work supporting your answer, use proper limit notation, and mention the test that you are using. [AJN]

$$\lim_{(x,y)\to(0,0)} \frac{xy}{x^2+y^2}$$

FORMULA SHEET

- $\langle u_1, u_2, u_3 \rangle \cdot \langle v_1, v_2, v_3 \rangle = u_1 v_1 + u_2 v_2 + u_3 v_3$
- $\mathbf{u} \cdot \mathbf{v} = |\mathbf{u}||\mathbf{v}| \cos(\theta)$
- $\langle u_1, u_2, u_3 \rangle \times \langle v_1, v_2, v_3 \rangle = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ u_1 & u_2 & u_3 \\ v_1 & v_2 & v_3 \end{vmatrix}$
- $|\mathbf{u} \times \mathbf{v}| = |\mathbf{u}||\mathbf{v}||\sin(\theta)|$
- $L = \int_a^b |\mathbf{r}'(t)| \, dt$
- $s(t) = \int_{t_0}^t |\mathbf{r}'(\tau)| \, d\tau$
- $\mathbf{T} = \dfrac{\mathbf{v}}{|\mathbf{v}|} = \dfrac{d\mathbf{r}}{ds}$
- $\kappa = \left| \dfrac{d\mathbf{T}}{ds} \right| = \dfrac{1}{|\mathbf{v}|} \left| \dfrac{d\mathbf{T}}{dt} \right| = \dfrac{|\mathbf{v} \times \mathbf{a}|}{|\mathbf{v}|^3}$
- $\mathbf{N} = \dfrac{1}{\kappa} \dfrac{d\mathbf{T}}{ds} = \dfrac{d\mathbf{T}/dt}{|d\mathbf{T}/dt|}$

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