**MATH 2551 GT-E Midterm 1**
**VERSION B**
**Summer 2025**
**COVERS SECTIONS 12.1-12.6, 13.1-13.4, 14.1-14.2**

**Full name:** _____ **GT ID:**_____

**Honor code statement:** I will abide strictly by the Georgia Tech honor code at all times. I will not use a calculator. I do not have a phone within reach, and I will not reference any website, application, or other CAS-enabled service. I will not consult with my notes or anyone during this exam. I will not provide aid to anyone else during this exam.

(  ) All of the knowledge presented in this exam is entirely my own. I am initialing to the left to attest to my integrity.

**Read all instructions carefully** before beginning.

- Print your name and GT ID neatly above.
- You have 75 minutes to take the exam.
- You may not use aids of any kind.
- Please show your work [J] and annotate your work using proper notation [N].
- Good luck!

| Question | Points |
|---|---|
| 1 | 2 |
| 2 | 2 |
| 3 | 4 |
| 4 | 10 |
| 5 | 10 |
| 6 | 10 |
| 7 | 6 |
| 8 | 6 |
| Total: | 50 |

For problems 1-2 choose whether each statement is true or false. If the statement is *always* true, pick true. If the statement is *ever* false, pick false. For all problems on this page please be sure to neatly fill in the bubble corresponding to your answer choice. [A]

1. (2 points) If $\mathbf{u}$ and $\mathbf{v}$ are vectors in $\mathbb{R}^3$, then $\mathbf{u} \times \mathbf{v} = \mathbf{v} \times \mathbf{u}$.

   ◯ **TRUE** ◯ **FALSE**

2. (2 points) The sphere $(x+1)^2 + (y-2)^2 + z^2 = 9$ is centered at $(1, -2, 0)$ and has radius $r = 3$.

   ◯ **TRUE** ◯ **FALSE**

3. (4 points) Which of the following vectors could be the principal unit normal vector at time $t = 2$ to a curve whose tangent line at $t = 2$ is given by

$$\ell(t) = \langle -1, 0, 0 \rangle + t\langle 1, -1, 1 \rangle.$$

   You must justify your answer in the space provided to receive full credit. [AJ]

   ◯ **A)** $\langle 1, 2, 1 \rangle$

   ◯ **B)** $\langle 0, 0, 1 \rangle$

   ◯ **C)** $\left\langle \frac{1}{3}, \frac{-2}{3}, \frac{2}{3} \right\rangle$

   ◯ **D)** $\left\langle \frac{1}{\sqrt{6}}, \frac{2}{\sqrt{6}}, \frac{1}{\sqrt{6}} \right\rangle$

   ◯ **E)** $\left\langle \frac{-2}{\sqrt{5}}, \frac{1}{\sqrt{5}}, 0 \right\rangle$

4. Let $P_1$ be the plane $x - 2y + 2z = 6$ and $P_2$ be the plane $2x - y + z = 6$. [AJN]

   (a) (2 points) Show that $P_1$ and $P_2$ are intersecting planes by showing that the point $(2, 2, 4)$ lies on both planes.

   (b) (2 points) Find normal vectors to both planes.

   (c) (6 points) Find a parameterization of the line of intersection of the planes.
   *Hint:* $\mathbf{v} = \mathbf{n}_1 \times \mathbf{n}_2$.

5. Satsuki is riding the Cat-Bus up a large hill in the village of Sayama Hills, and the path the bus takes up the hill is given by $\mathbf{r}(t) = \langle \cos 2t, \sin 2t, t \rangle$, for $0 \leq t < 2\pi$, where $t$ is measured in minutes and $\mathbf{r}(t)$ in kilometers. [AJN]

   (a) (2 points) What was Satsuki's speed halfway through her journey, when $t = \pi$?

   (b) (2 points) What is Satsuki's **position** at the end of her journey, when $t = 2\pi$?

   (c) (6 points) How far did Satsuki travel in total from time $t = 0$ to time $t = 2\pi$?

6. In this problem, you will work with the curve

$$\mathbf{r}(t) = t\mathbf{i} + (\ln \cos t)\mathbf{j}$$

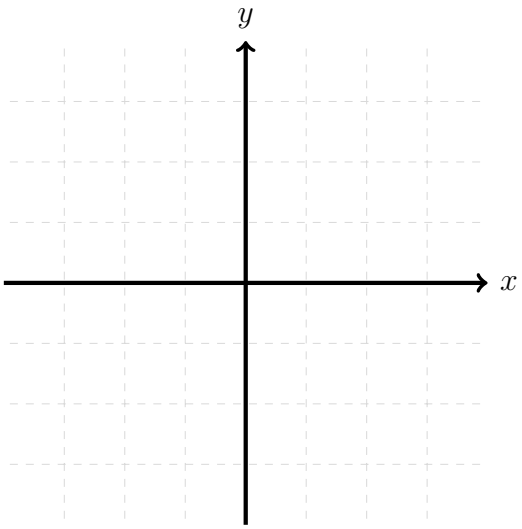
for $-\pi/2 < t < \pi/2$. [AJN]

(a) (5 points) Compute the unit tangent vector $\mathbf{T}(t)$. *Hint: chain rule.*

(b) (3 points) Compute the principal unit normal vector $\mathbf{N}(t)$.

(c) (2 points) Compute the curvature $\kappa(t)$.

7. (6 points) Let $f(x, y) = \sqrt{y - x}$. Graph the domain of $f$ on the provided axes below, and clearly label the axes. Be sure to show all your work in finding the domain. Indicate whether or not each part of the boundary of the domain is included. [AJN]



8. (6 points) The limit below exists. Evaluate the limit. To receive full credit, you must show work supporting your answer and use proper notation. *Hint: use algebra.* [AJN]

$$\lim_{(x,y)\to(4,3)} \frac{\sqrt{x}-\sqrt{y+1}}{x-y-1}$$

## FORMULA SHEET

- $\langle u_1, u_2, u_3\rangle \cdot \langle v_1, v_2, v_3\rangle = u_1v_1 + u_2v_2 + u_3v_3$
- $\mathbf{u} \cdot \mathbf{v} = |\mathbf{u}||\mathbf{v}| \cos(\theta)$
- $\langle u_1, u_2, u_3\rangle \times \langle v_1, v_2, v_3\rangle = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ u_1 & u_2 & u_3 \\ v_1 & v_2 & v_3 \end{vmatrix}$
- $|\mathbf{u} \times \mathbf{v}| = |\mathbf{u}||\mathbf{v}||\sin(\theta)|$
- $L = \displaystyle\int_a^b |\mathbf{r}'(t)|\, dt$
- $s(t) = \displaystyle\int_{t_0}^t |\mathbf{r}'(\tau)|\, d\tau$
- $\mathbf{T} = \dfrac{\mathbf{v}}{|\mathbf{v}|} = \dfrac{d\mathbf{r}}{ds}$
- $\kappa = \left|\dfrac{d\mathbf{T}}{ds}\right| = \dfrac{1}{|\mathbf{v}|}\left|\dfrac{d\mathbf{T}}{dt}\right| = \dfrac{|\mathbf{v} \times \mathbf{a}|}{|\mathbf{v}|^3}$
- $\mathbf{N} = \dfrac{1}{\kappa}\dfrac{d\mathbf{T}}{ds} = \dfrac{d\mathbf{T}/dt}{|d\mathbf{T}/dt|}$

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