MATH 2551 GT-E Midterm 1 Make-up
VERSION C
Summer 2025
COVERS SECTIONS 12.1-12.6, 13.1-13.4, 14.1-14.2

Full name: ______________________________ **GT ID:**____________

Honor code statement: I will abide strictly by the Georgia Tech honor code at all times. I will not use a calculator. I do not have a phone within reach, and I will not reference any website, application, or other CAS-enabled service. I will not consult with my notes or anyone during this exam. I will not provide aid to anyone else during this exam.

(   ) All of the knowledge presented in this exam is entirely my own. I am initialing to the left to attest to my integrity.

Read all instructions carefully before beginning.

- Print your name and GT ID neatly above.
- You have 75 minutes to take the exam.
- You may not use aids of any kind.
- Please show your work [J] and annotate your work using proper notation [N].
- Good luck!

Question	Points
1	2
2	2
3	2
4	10
5	10
6	12
7	6
8	6
Total:	50

For problems 1-2 choose whether each statement is true or false. If the statement is *always* true, pick true. If the statement is *ever* false, pick false. For all problems on this page please be sure to neatly fill in the bubble corresponding to your answer choice. [A]

1. (2 points) If $\mathbf{n} = \mathbf{u} \times \mathbf{v}$, then $(\mathbf{u} \cdot \mathbf{n})^2 + (\mathbf{v} \cdot \mathbf{n})^2 = 0$.

   ○ **TRUE** ○ **FALSE**

2. (2 points) If a fly is buzzing around the room along a curve with arc-length parameterization $r(s) = \left\langle \cos\left(\frac{s}{\sqrt{2}}\right), \sin\left(\frac{s}{\sqrt{2}}\right), \frac{s}{\sqrt{2}} \right\rangle$, $0 \leq s \leq 2\pi$ measured in meters, then when the fly is located at $r(\pi)$ the fly has travelled π meters.

   ○ **TRUE** ○ **FALSE**

3. (2 points) Which plane given below is **not** parallel to the plane $x - 2y + 2z = 4$? [A]

   ○ **A)** $x - 2y + 2z = 6$

   ○ **B)** $6x - 12y + 12z = 0$

   ○ **C)** The plane through the origin which is orthogonal to $\langle -1, 2, -2 \rangle$

   ○ **D)** The plane containing the line $\ell_1(t) = t\langle 1, -2, 2 \rangle$ and the line $\ell_2(t) = t\langle 4, 1, -1 \rangle$.

   ○ **E)** The plane containing the points $(2, 0, -1)$, $(2, 1, 0)$, and $(4, 1, -1)$.

4. (10 points) Let P_1 be the plane defined by the equation $2x - 2y + z = 1$ and let Q be the point $(1, 2, 1)$. [AJN]

   (a) Find an equation for the plane P_2 which contains the point Q and is parallel to P_1.

   (b) Find an equation for the line ℓ which passes through Q and is orthogonal to both planes.

   (c) Find the point R where the line ℓ intersects the plane P_1.

5. (10 points) Find the coordinates of the point which is a distance of $\sqrt{5}\pi/2$ along the helix $\mathbf{r}(t) = \langle \cos 2t, \sin 2t, t \rangle$ in the direction of increasing parameter t from $(2, 0, 0)$. *Hint: find the arc-length parameter* $s(t) = \int_{t_0}^{t} |\mathbf{r}'(\tau)|\, d\tau$ *for a good choice of* t_0. [AJN]

6. (12 points) In this problem, you will work with the helix curve

$$\mathbf{r}(t) = \langle 3\cos t, 3\sin t, 4t\rangle$$

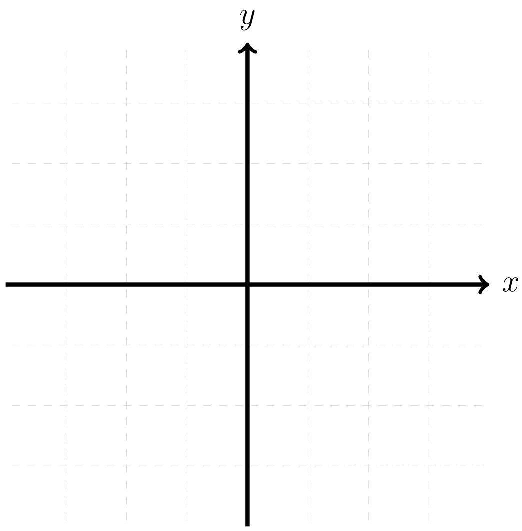
for $0 \leq t \leq \pi$. [AJN]

(a) Compute the unit tangent vector $\mathbf{T}(t)$.

(b) Compute the principal unit normal vector $\mathbf{N}(t)$.

(c) Compute the curvature $\kappa(t)$.

7. (6 points) Let $f(x,y) = \frac{1}{\sqrt{9-x^2-y^2}}$. Graph the domain of f on the provided axes below, and clearly label the axes. Be sure to show all your work in finding the domain. Indicate whether or not each part of the boundary of the domain is included. [AJN]



8. (6 points) Find a value C so that the function $h(x,y) = \dfrac{3x^2}{x^2+y^2}$ satisfies $|h(x,y)| \leq C$. Then, use the Squeeze Theorem to show that

$$\lim_{(x,y)\to(0,0)} \frac{3x^2y^3}{x^2y^2+y^4} = 0.$$

[AJN]

FORMULA SHEET

- $\langle u_1, u_2, u_3 \rangle \cdot \langle v_1, v_2, v_3 \rangle = u_1 v_1 + u_2 v_2 + u_3 v_3$
- $\mathbf{u} \cdot \mathbf{v} = |\mathbf{u}||\mathbf{v}| \cos(\theta)$
- $\langle u_1, u_2, u_3 \rangle \times \langle v_1, v_2, v_3 \rangle = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ u_1 & u_2 & u_3 \\ v_1 & v_2 & v_3 \end{vmatrix}$
- $|\mathbf{u} \times \mathbf{v}| = |\mathbf{u}||\mathbf{v}||\sin(\theta)|$
- $L = \displaystyle\int_a^b |\mathbf{r}'(t)| \, dt$
- $s(t) = \displaystyle\int_{t_0}^t |\mathbf{r}'(\tau)| \, d\tau$
- $\mathbf{T} = \dfrac{\mathbf{v}}{|\mathbf{v}|} = \dfrac{d\mathbf{r}}{ds}$
- $\kappa = \left|\dfrac{d\mathbf{T}}{ds}\right| = \dfrac{1}{|\mathbf{v}|}\left|\dfrac{d\mathbf{T}}{dt}\right| = \dfrac{|\mathbf{v} \times \mathbf{a}|}{|\mathbf{v}|^3}$
- $\mathbf{N} = \dfrac{1}{\kappa}\dfrac{d\mathbf{T}}{ds} = \dfrac{d\mathbf{T}/dt}{|d\mathbf{T}/dt|}$

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