

**MATH 2551 GT-E Midterm 2**  
**VERSION A**  
**Summer 2025**  
**COVERS SECTIONS 14.3-14.8, 15.1-15.4**

**Full name:** \_\_\_\_\_ **GT ID:** \_\_\_\_\_

**Honor code statement:** I will abide strictly by the Georgia Tech honor code at all times. I will not use a calculator. I do not have a phone within reach, and I will not reference any website, application, or other CAS-enabled service. I will not consult with my notes or anyone during this exam. I will not provide aid to anyone else during this exam.

(     ) All of the knowledge presented in this exam is entirely my own. I am initialing to the left to attest to my integrity.

**Read all instructions carefully** before beginning.

- Print your name and GT ID neatly above.
- You have 75 minutes to take the exam.
- You may not use aids of any kind.
- Please show your work [J] and annotate your work using proper notation [N].
- Good luck!

Question	Points
1	2
2	2
3	2
4	2
5	5
6	5
7	4
8	8
9	10
10	10
Total:	50

For problems 1-2 choose whether each statement is true or false. If the statement is *always* true, pick true. If the statement is *ever* false, pick false. For all problems on this page please be sure to neatly fill in the bubble corresponding to your answer choice. [A]

1. (2 points) The function  $f(x, y) = e^{2x} \cos y$  has no critical points.

☐ **TRUE**

☐ **FALSE**

2. (2 points) Suppose  $f(x, y, z)$  is a differentiable function at  $P(x_P, y_P, z_P)$ . State an equation for the linearization  $L(x, y, z)$  of  $f$  at  $P$ . [AN]

3. (2 points) Suppose  $w = f(x, y, z)$  is a differentiable function and  $x(u, v), y(u, v), z(u, v)$  are all differentiable. Give the formula for  $\frac{\partial w}{\partial v}$ . [AN]

4. (2 points) Which of the following is FALSE about the function  $f(x, y) = \begin{cases} 1 & xy = 0 \\ 0 & xy \neq 0 \end{cases}$ .  
Select all that apply. [A]

☐ **A)**  $f_x$  is continuous at  $(0, 0)$ .

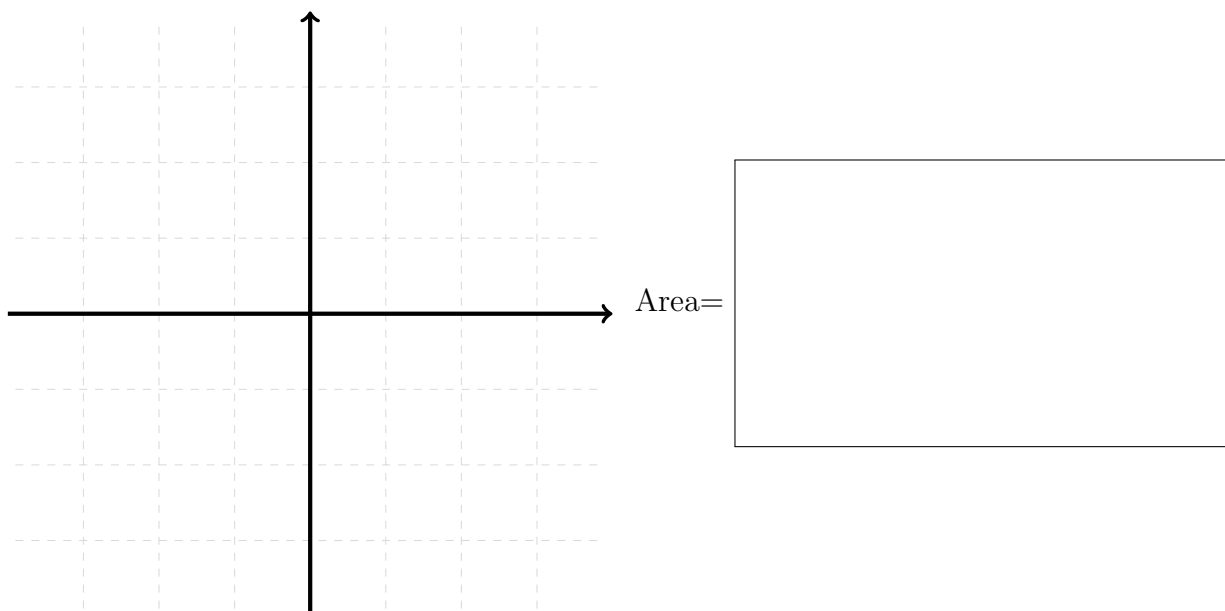
☐ **B)**  $f_x$  is continuous at  $(1, 1)$ .

☐ **C)**  $f_x$  is continuous at  $(1, 0)$ .

☐ **D)**  $f$  is differentiable at  $(0, 0)$ .

☐ **E)**  $f$  is differentiable at  $(1, 1)$ .

5. (5 points) (a) Sketch the region  $R$  on the axes provided which is bounded by the curve  $y = e^x$  and the lines  $y = 2$  and  $y = 1 - x$ . Include labels for the axes, labels for the boundary curves or lines, and labels for the points of intersection. [AN]
- (b) Set up *but do not evaluate* a single iterated integral which computes the area of the region. Put your answer in the box. [AN]



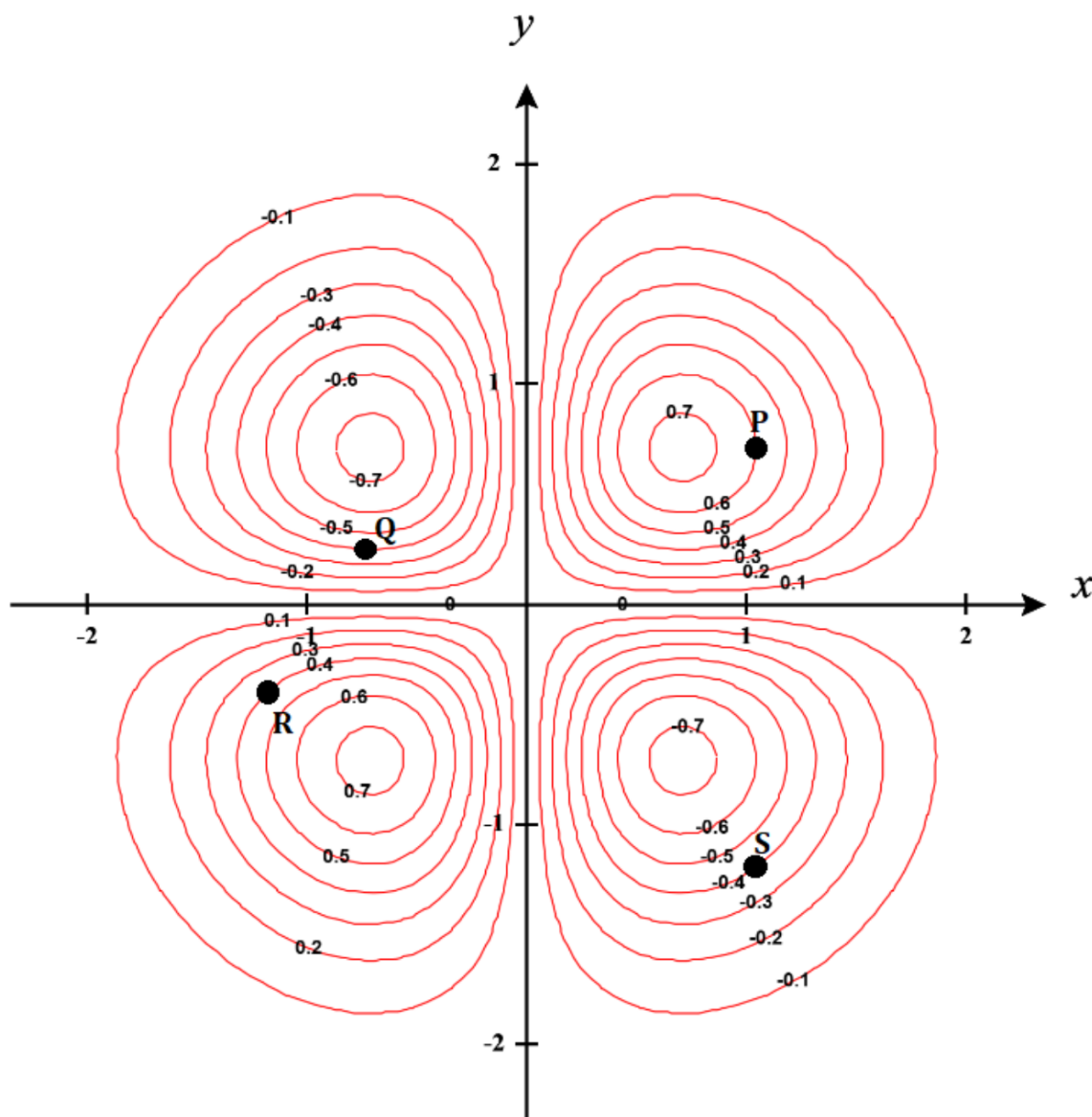
6. (5 points) Convert the given integral to polar coordinates. *Do not evaluate!* [AN]

$$\int_0^2 \int_0^{\sqrt{2x-x^2}} \frac{1}{(x^2 + y^2)^2} dy dx$$

Volume=

7. In this problem, you will work with the function  $f : \mathbb{R}^2 \rightarrow \mathbb{R}$  whose contour plot near the origin is shown below. [AN]

- (a) (1 point) Determine the sign  $(+, -, 0)$  of the directional derivative at the point  $Q$  in the direction  $\langle 1, 1 \rangle$ .
- (b) (1 point) Determine the sign  $(+, -, 0)$  of the directional derivative at the point  $P$  in the direction  $\langle 0, -1 \rangle$ .
- (c) (2 points) Draw a vector which is a positive scalar multiple of the gradient vector of  $f$  at the point  $R$ .



8. (8 points) Let  $f : \mathbb{R}^3 \rightarrow \mathbb{R}$  be the function  $f(x, y, z) = (x - z)y$  and  $\mathbf{r}(s, t) : \mathbb{R}^2 \rightarrow \mathbb{R}^3$  be the function

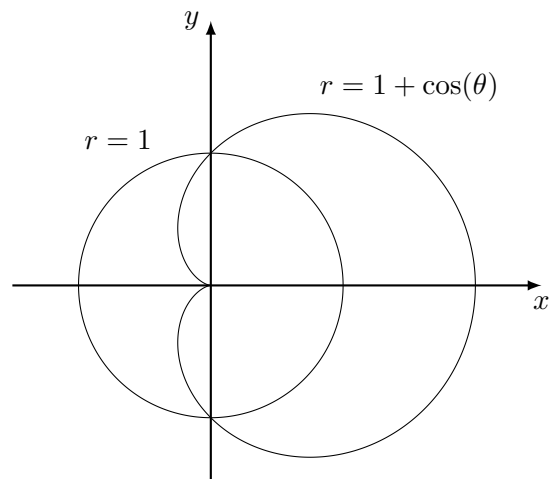
$$\mathbf{r}(s, t) = \begin{bmatrix} s - 2 \\ st \\ e^{st} \end{bmatrix}.$$

- (a) Find  $Df$ .
- (b) Find  $D\mathbf{r}$ .
- (c) Evaluate  $\mathbf{r}(3, 1)$  and  $D\mathbf{r}|_{(s,t)=(3,1)}$ .
- (d) Finally, use the Chain Rule to evaluate  $D(f(\mathbf{r}(s, t)))|_{(s,t)=(3,1)}$ .

[AJN]

9. (10 points) (a) Write and evaluate the integral for the area of the region  $R$  between the cardioid  $r = 1 + \cos(\theta)$  and the circle  $r = 1$  which is completely contained in Quadrants I and IV. (b) Then, use the fact that  $\iint_R f(x, y) \, dA = 5\pi/8$  to find the average value of  $f(x, y) = x$  over the region  $R$ . *Hint:*  $\cos^2 t = \frac{1 + \cos 2t}{2}$ .

[AJN]  
[AJN]



10. (10 points) Find all critical points and use the Hessian matrix to classify the critical points of the function. [AJN]

$$f(x, y) = x^2 + y^3 - 3y$$

# FORMULA SHEET

- Total Derivative: For  $\mathbf{f}(x_1, \dots, x_n) = \langle f_1(x_1, \dots, x_n), \dots, f_m(x_1, \dots, x_n) \rangle$

$$D\mathbf{f} = \begin{bmatrix} (f_1)_{x_1} & (f_1)_{x_2} & \cdots & (f_1)_{x_n} \\ (f_2)_{x_1} & (f_2)_{x_2} & \cdots & (f_2)_{x_n} \\ \vdots & \ddots & \cdots & \vdots \\ (f_m)_{x_1} & (f_m)_{x_2} & \cdots & (f_m)_{x_n} \end{bmatrix}$$

- Linearization: Near  $\mathbf{a}$ ,  $f(\mathbf{x}) \approx L(\mathbf{x}) = f(\mathbf{a}) + Df(\mathbf{a})(\mathbf{x} - \mathbf{a})$
- Chain Rule: If  $h = g(f(\mathbf{x}))$  then  $Dh(\mathbf{x}) = Dg(f(\mathbf{x}))Df(\mathbf{x})$
- Implicit Differentiation: If  $z$  is implicitly given in terms of  $x$  and  $y$  by  $F(x, y, z) = c$ , then  $\frac{\partial z}{\partial x} = \frac{-F_x}{F_z}$  and  $\frac{\partial z}{\partial y} = \frac{-F_y}{F_z}$ .
- Directional Derivative: If  $\mathbf{u}$  is a unit vector,  $D_{\mathbf{u}}f(P) = Df(P)\mathbf{u} = \nabla f(P) \cdot \mathbf{u}$
- The tangent line to a level curve of  $f(x, y)$  at  $(a, b)$  is  $0 = \nabla f(a, b) \cdot \langle x - a, y - b \rangle$
- The tangent plane to a level surface of  $f(x, y, z)$  at  $(a, b, c)$  is

$$0 = \nabla f(a, b, c) \cdot \langle x - a, y - b, z - c \rangle.$$

- Hessian Matrix: For  $f(x, y)$ ,  $Hf(x, y) = \begin{bmatrix} f_{xx} & f_{yx} \\ f_{xy} & f_{yy} \end{bmatrix}$
- Second Derivative Test: If  $(a, b)$  is a critical point of  $f(x, y)$  then
  1. If  $\det(Hf(a, b)) > 0$  and  $f_{xx}(a, b) < 0$  then  $f$  has a local maximum at  $(a, b)$
  2. If  $\det(Hf(a, b)) > 0$  and  $f_{xx}(a, b) > 0$  then  $f$  has a local minimum at  $(a, b)$
  3. If  $\det(Hf(a, b)) < 0$  then  $f$  has a saddle point at  $(a, b)$
  4. If  $\det(Hf(a, b)) = 0$  the test is inconclusive

- Area/volume:  $\text{area}(R) = \iint_R dA$

- Trig identities:  $\sin^2(x) = \frac{1}{2}(1 - \cos(2x))$ ,  $\cos^2(x) = \frac{1}{2}(1 + \cos(2x))$

- Average value:  $f_{\text{avg}} = \frac{\iint_R f(x, y) dA}{\text{area of } R}$

- Polar coordinates:  $x = r \cos(\theta)$ ,  $y = r \sin(\theta)$ ,  $dA = r \, dr \, d\theta$



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