MATH 2551 GT-E Midterm 2 Make-up
VERSION C
Summer 2025
COVERS SECTIONS 14.3-14.8, 15.1-15.4

Full name: ______________________________ **GT ID:**____________

Honor code statement: I will abide strictly by the Georgia Tech honor code at all times. I will not use a calculator. I do not have a phone within reach, and I will not reference any website, application, or other CAS-enabled service. I will not consult with my notes or anyone during this exam. I will not provide aid to anyone else during this exam.

(   ) All of the knowledge presented in this exam is entirely my own. I am initialing to the left to attest to my integrity.

Read all instructions carefully before beginning.

- Print your name and GT ID neatly above.
- You have 75 minutes to take the exam.
- You may not use aids of any kind.
- Please show your work [J] and annotate your work using proper notation [N].
- Good luck!

Question	Points
1	2
2	2
3	2
4	3
5	5
6	5
7	7
8	8
9	8
10	8
Total:	50

1. (2 points) *Fill in the blanks.* If the region R is [ ] and [ ], and the function $z = f(x, y)$ is [ ] on R, then f attains an absolute maximum and absolute minimum value on R. [AN]

2. (2 points) Suppose $f(x, y)$ is a differentiable function at $P(x_P, y_P)$. State an equation for the linearization $L(x, y)$ of f at P. [AN]

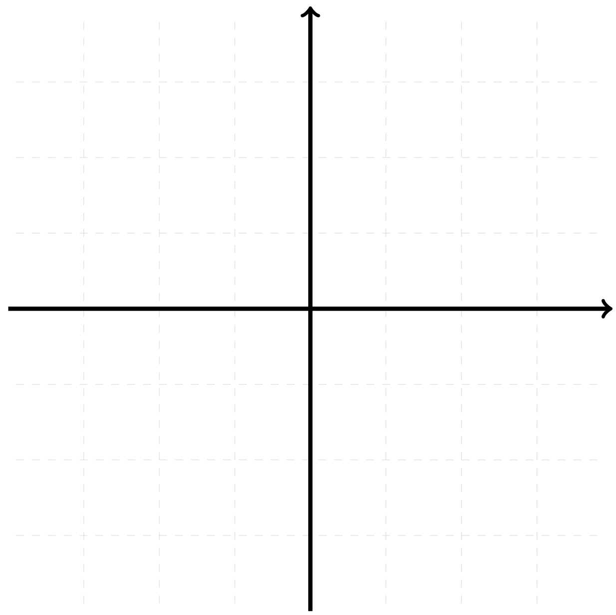
3. (2 points) Suppose $z = f(x, y)$ is a differentiable function and $x(t), y(t)$ are all differentiable. Give the formula for $\frac{dz}{dt}$. [AN]

4. (3 points) Suppose $f(x, y) = \begin{cases} 1, & \text{if } xy = 0 \\ 0, & \text{if } xy \neq 0 \end{cases}$. Fill in the boxes below, or enter DNE in the box if there is no value. [AN]

$f(0,0) =$	$f(1,0) =$	$f(1,1) =$
$f_x(0,0) =$	$f_x(1,0) =$	$f_x(1,1) =$
$f_y(1,0) =$	$f_y(0,1) =$	$f_y(-1,1) =$

5. (5 points) (a) Sketch the region R on the axes provided which is bounded by $y = x - 2$, $y = -x$, and the curve $y = \sqrt{x}$. Include labels for the axes, labels for the boundary curves or lines, and the points of intersection. [AN]

(b) Set up *but do not evaluate* a sum of iterated integrals which computes the area of the region. Put your answer in the box. [AN]



Area=

6. (5 points) Convert the given integral to polar coordinates. *Do not evaluate!* [AN]

$$\int_0^2 \int_0^x y \, dy \, dx$$

Volume=

7. (7 points) Use the function $z = f(x, y)$ and the point P to answer the questions below. [AJN]

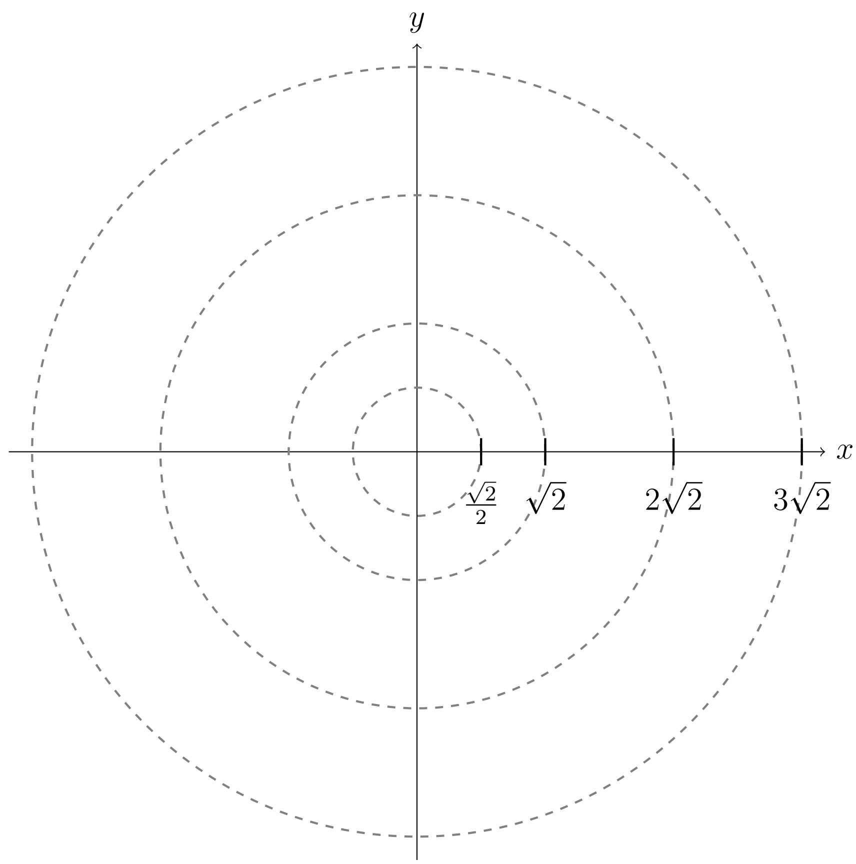
$$f(x, y) = \ln(x^2 + y^2) \text{ and } P(1, 1)$$

(a) Find the directional derivative $D_{\mathbf{u}}f$ of f in the direction of $\mathbf{u} = \langle 4, 5\rangle$ at $P(1, 1)$.

(b) Find a vector $\mathbf{v}$ which points in the direction which *minimizes* the value of $D_{\mathbf{u}}f$ at $P(1, 1)$.

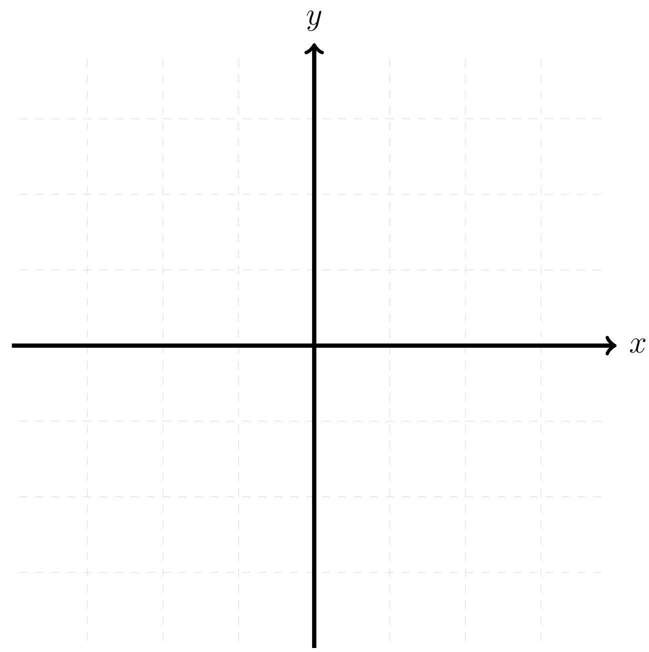
(c) Find the gradient ∇f of f.

(d) Sketch $P(1, 1)$ and the gradient vector $\nabla f|_{(x,y)=(1,1)}$ on the axes provided.

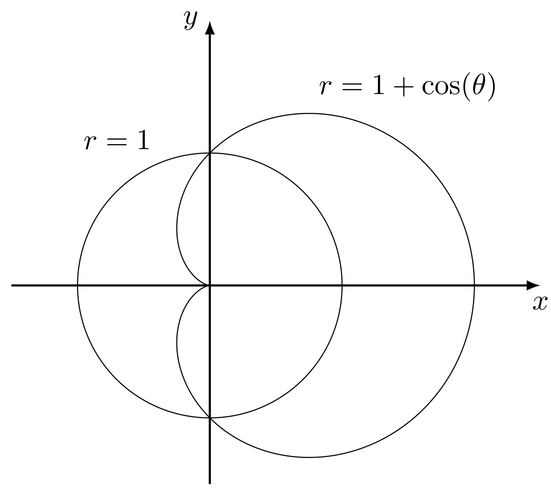


8. (8 points) Sketch the domain R on the axes provided, and find the absolute maxima and minima of the function $f(x, y)$ on R. [AJN]

$$f(x, y) = x^2 + xy + y^2, \text{ and } R \text{ is the region bounded by } y = x^2 \text{ and } y = 1.$$



9. (8 points) (a) Write and evaluate the integral for the area of the region R between the cardioid $r = 1 + \cos(\theta)$ and the circle $r = 1$ which is completely contained in Quadrants I and IV. (b) Then, use the fact that $\iint_R f(x, y)\, dA = 5\pi/8$ to find the average value of $f(x, y) = x$ over the region R. *Hint:* $\cos^2 t = \frac{1+\cos 2t}{2}$. [AJN]



10. (8 points) Find the Hessian matrix Hf and use the Hessian matrix to classify the critical points $(0, 0)$ and $(-2, 0)$ of the function $z = f(x, y)$. You do not need to find or classify any other critical points of $f(x, y)$. [AJN]

$$f(x, y) = x^3 - 3xy^2 + 3x^2 + 3y^2$$

FORMULA SHEET

- Total Derivative: For $\mathbf{f}(x_1, \ldots, x_n) = \langle f_1(x_1, \ldots, x_n), \ldots, f_m(x_1, \ldots, x_n) \rangle$

$$D\mathbf{f} = \begin{bmatrix} (f_1)_{x_1} & (f_1)_{x_2} & \cdots & (f_1)_{x_n} \\ (f_2)_{x_1} & (f_2)_{x_2} & \cdots & (f_2)_{x_n} \\ \vdots & \ddots & \cdots & \vdots \\ (f_m)_{x_1} & (f_m)_{x_2} & \cdots & (f_m)_{x_n} \end{bmatrix}$$

- Linearization: Near $\mathbf{a}$, $f(\mathbf{x}) \approx L(\mathbf{x}) = f(\mathbf{a}) + Df(\mathbf{a})(\mathbf{x} - \mathbf{a})$

- Chain Rule: If $h = g(f(\mathbf{x}))$ then $Dh(\mathbf{x}) = Dg(f(\mathbf{x}))Df(\mathbf{x})$

- Implicit Differentiation: If z is implicitly given in terms of x and y by $F(x, y, z) = c$, then $\dfrac{\partial z}{\partial x} = \dfrac{-F_x}{F_z}$ and $\dfrac{\partial z}{\partial y} = \dfrac{-F_y}{F_z}$.

- Directional Derivative: If $\mathbf{u}$ is a unit vector, $D_{\mathbf{u}}f(P) = Df(P)\mathbf{u} = \nabla f(P) \cdot \mathbf{u}$

- The tangent line to a level curve of $f(x, y)$ at (a, b) is $0 = \nabla f(a, b) \cdot \langle x - a, y - b \rangle$

- The tangent plane to a level surface of $f(x, y, z)$ at (a, b, c) is

$$0 = \nabla f(a, b, c) \cdot \langle x - a, y - b, z - c \rangle.$$

- Hessian Matrix: For $f(x, y)$, $Hf(x, y) = \begin{bmatrix} f_{xx} & f_{yx} \\ f_{xy} & f_{yy} \end{bmatrix}$

- Second Derivative Test: If (a, b) is a critical point of $f(x, y)$ then
  1. If $\det(Hf(a, b)) > 0$ and $f_{xx}(a, b) < 0$ then f has a local maximum at (a, b)
  2. If $\det(Hf(a, b)) > 0$ and $f_{xx}(a, b) > 0$ then f has a local minimum at (a, b)
  3. If $\det(Hf(a, b)) < 0$ then f has a saddle point at (a, b)
  4. If $\det(Hf(a, b)) = 0$ the test is inconclusive

- Area/volume: $\text{area}(R) = \iint_R dA$

- Trig identities: $\sin^2(x) = \frac{1}{2}(1 - \cos(2x)), \quad \cos^2(x) = \frac{1}{2}(1 + \cos(2x))$

- Average value: $f_{avg} = \dfrac{\iint_R f(x, y) dA}{\text{area of } R}$

- Polar coordinates: $x = r\cos(\theta), \quad y = r\sin(\theta), \quad dA = r\ dr\ d\theta$

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