MATH 2551 GT-E Midterm 3
VERSION A
Summer 2025
COVERS SECTIONS 15.5-15.8, 16.1-16.8

Full name: ______________________________ **GT ID:**______________

Honor code statement: I will abide strictly by the Georgia Tech honor code at all times. I will not use a calculator. I do not have a phone within reach, and I will not reference any website, application, or other CAS-enabled service. I will not consult with my notes or anyone during this exam. I will not provide aid to anyone else during this exam.

(   ) All of the knowledge presented in this exam is entirely my own. I am initialing to the left to attest to my integrity.

Read all instructions carefully before beginning.

- Print your name and GT ID neatly above.
- You have 75 minutes to take the exam.
- You may not use aids of any kind.
- Please show your work [J] and annotate your work using proper notation [N].
- Good luck!

Question	Points
1	2
2	2
3	2
4	10
5	6
6	10
7	8
8	10
Total:	50

For problems 1-2 choose whether each statement is true or false. If the statement is *always* true, pick true. If the statement is *ever* false, pick false. For all problems on this page please be sure to neatly fill in the bubble corresponding to your answer choice. [A]

1. (2 points) If $\mathbf{F}$ is any vector field and C is a closed simple loop in $\mathbb{R}^2$, then the circulation of $\mathbf{F}$ around C traversed in one orientation is the negative of the circulation of $\mathbf{F}$ around C traversed in the opposite orientation.

   ◯ **TRUE** ◯ **FALSE**

2. (2 points) If C is a curve with parametrization $\mathbf{r}_1(t) = \langle x(t), y(t) \rangle$ with $t \in [a, b]$, then the parametrization $\mathbf{r}_2(t) = \mathbf{r}_1(-t)$ with $t \in [-b, -a]$ is also a parametrization of C with the opposite orientation. [A]

   ◯ **TRUE** ◯ **FALSE**

3. (2 points) If $g(x, y)$ is strictly positive at all points in $\mathbb{R}^2$, then

$$\iint_{R_1} g(x,y)\,dA \geq \iint_{R_2} g(x,y)\,dA$$

   if the area of R_1 is larger than the area of R_2. [A]

   ◯ **TRUE** ◯ **FALSE**

4. (10 points) In this problem you will compute the integral

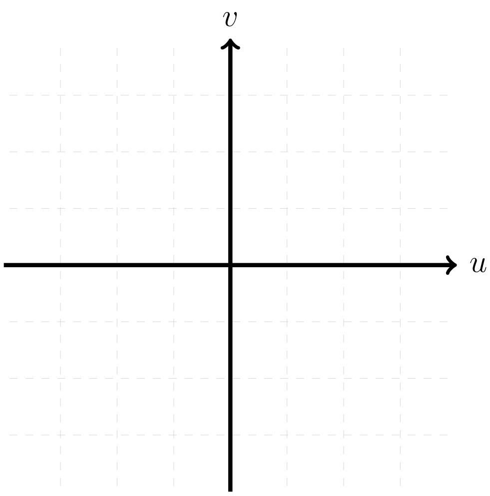
$$M = \iint_R (x+y)e^{x^2-y^2}\,dx\,dy$$

for the region R in the first quadrant bounded by the lines $x-y=0$, $x-y=3$, $x+y=1$, $x+y=3$ using the transformation $u=x-y$, $v=x+y$.

(a) On the axes provided, sketch the new region of integration G in the uv-plane. [A]

(b) Solve the transformation equations for x and y in terms of u and v and compute the Jacobian determinant, i.e., find $\mathbf{T}(u,v)$ and $|\det D\mathbf{T}(u,v)|$. [AJN]

(c) Evaluate the double integral using a change of variables and your results from parts (a) and (b) to find the value of M. *Hint: integrate with the order $du\,dv$.* [AJN]



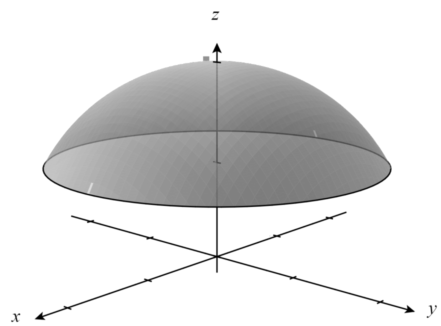
$\mathbf{T}(u,v)=$

$|\det D\mathbf{T}(u,v)|=$

5. (6 points) Let D be the smaller cap cut from a solid ball of radius 1 units by the plane $z = 1/2$. The point $P(0, \frac{\sqrt{3}}{2}, \frac{1}{2})$ is on D at the intersection of the ball and the plane.

   (a) Find the spherical coordinates (ρ, φ, θ) of the point $P(0, \frac{\sqrt{3}}{2}, \frac{1}{2})$. [AN]

   (b) Express the volume of D as an iterated triple integral in spherical coordinates. *Do not evaluate!* [AN]



$(0, \frac{\sqrt{3}}{2}, \frac{1}{2})_{\mathcal{S}}=$

Volume=

6. (10 points) This problem will have you compute the flux of the vector field $\mathbf{F}(x, y, z) = x\mathbf{i} + y\mathbf{j}$ through S the open ended circular cylinder of radius 3 and height 5 with its base on the xy-plane and centered about the z-axis, oriented away from the z-axis. [AJN]

   (a) Set up and evaluate a surface integral which computes the flux of $\mathbf{F}$ through S.

   (b) Why can we NOT evaluate a line integral over the boundary C to obtain the result of part (a) using Stokes' Theorem? Use at least one or two complete sentences to answer.

7. (8 points) Consider the vector field $\mathbf{F} = \langle zx, zy, z \rangle$ and the surface S consisting of the portion of the paraboloid $z = 9 - x^2 - y^2$ with $z \geq 0$ together with the circular disk $x^2 + y^2 \leq 9$ in the xy-plane, oriented with normal vectors away from the origin. [AJN]

   (a) Use the Divergence Theorem to set up a triple iterated integral over the region D which is enclosed by S, which evaluates to the same value as $\iint_S \mathbf{F} \cdot \mathbf{n} \, d\sigma$.
   Do not evaluate!

   (b) Is there a vector field $\mathbf{G}$ such that $\mathbf{F} = \nabla \times \mathbf{G}$? Explain in one or two complete sentences how you know.

8. (10 points) Consider the vector field $\mathbf{F} = \langle -y, 2x \rangle$ and the curve C which is the circle $x^2 + y^2 = 25$ oriented counterclockwise with outward pointing normal vector. [AJN]

   (a) Compute the flow of F around T, which is $\int_C F \cdot \mathbf{T}\, ds$, **with or without** using Green's Theorem. *Use either method, but you can use the other integral to check your answer.*

   (b) Compute the flux of F around T, which is $\int_C F \cdot \mathbf{n}\, ds$, **using** Green's Theorem.

   (c) Is F conservative? Explain in at least one or two complete sentences how you know.

If you need more room, please use the next page and leave a note at the bottom of this page.

If you need more room on the previous problem please use this page and indicate it at the bottom of the previous page.

FORMULA SHEET

- Trig identities: $\sin^2(x) = \frac{1}{2}(1 - \cos(2x))$, $\cos^2(x) = \frac{1}{2}(1 + \cos(2x))$
- Volume$(D) = \iiint_D dV$, $\quad f_{avg} = \dfrac{\iiint_D f(x,y,z)dV}{\text{volume of } D}$ or $\dfrac{\int_C f(x,y,z)\ ds}{\text{length of } C}$,
  Mass: $M = \iiint_D \delta\ dV$
- Cylindrical coordinates: $x = r\cos(\theta)$, $\quad y = r\sin(\theta)$, $z = z$, $dV = r\ dz\ dr\ d\theta$
- Spherical coordinates: $x = \rho\sin(\phi)\cos(\theta)$, $y = \rho\sin(\phi)\sin(\theta)$, $z = \rho\cos(\phi)$, $dV = \rho^2\sin(\phi)\ d\rho\ d\phi\ d\theta$
- Substitution for double integrals: If R is the image of G under a coordinate transformation $\mathbf{T}(u,v) = \langle x(u,v), y(u,v)\rangle$ then

$$\iint_R f(x,y)\ dx\ dy = \iint_G f(\mathbf{T}(u,v))|\det D\mathbf{T}(u,v)|\ du\ dv.$$

- Scalar line integral: $\int_C f(x,y,z)\ ds = \int_a^b f(\mathbf{r}(t))|\mathbf{r}'(t)|dt$
- Tangential vector line integral: $\int_C \mathbf{F}\cdot\mathbf{T}\ ds = \int_C \mathbf{F}\cdot d\mathbf{r} = \int_a^b \mathbf{F}(\mathbf{r}(t))\cdot\mathbf{r}'(t)\ dt$
- Normal vector line integral: $\int_C \mathbf{F}(x,y)\cdot\mathbf{n}\ ds = \int_C P\ dy - Q\ dx = \int_a^b \mathbf{F}(\mathbf{r}(t))\cdot\langle y'(t), -x'(t)\rangle\ dt$.
- Fundamental Theorem of Line Integrals: $\int_C \nabla f\cdot d\mathbf{r} = f(B) - f(A)$ if C is a path from A to B
- Curl (Mixed Partials) Test: $\mathbf{F} = \nabla f$ if curl $\mathbf{F} = \mathbf{0} \Leftrightarrow P_z = R_x, Q_z = R_y$, and $Q_x = P_y$.
- $\nabla = \left\langle \dfrac{\partial}{\partial x}, \dfrac{\partial}{\partial y}, \dfrac{\partial}{\partial z}\right\rangle$ $\qquad$ div $\mathbf{F} = \nabla\cdot\mathbf{F}$ $\qquad$ curl $\mathbf{F} = \nabla\times\mathbf{F}$
- Green's Theorem: If C is a simple closed curve with positive orientation and R is the simply-connected region it encloses, then

$$\int_C \mathbf{F}\cdot\mathbf{T}\ ds = \iint_R (\nabla\times\mathbf{F})\cdot\mathbf{k}\ dA \qquad \int_C \mathbf{F}\cdot\mathbf{n}\ ds = \iint_R (\nabla\cdot\mathbf{F})\ dA.$$

- Surface Area$=\iint_S 1\ d\sigma$
- Scalar surface integral: $\iint_S f(x,y,z)\ d\sigma = \iint_R f(\mathbf{r}(u,v))\ |\mathbf{r}_u\times\mathbf{r}_v|dA$
- Flux surface integral: $\iint_S \mathbf{F}\cdot\mathbf{n}\ d\sigma = \iint_S \mathbf{F}\cdot d\boldsymbol{\sigma} = \iint_R \mathbf{F}(\mathbf{r}(u,v))\cdot(\mathbf{r}_u\times\mathbf{r}_v)\ dA$
- Stokes' Theorem: If S is a piecewise smooth oriented surface bounded by a piecewise smooth curve C and $\mathbf{F}$ is a vector field whose components have continuous partial derivatives on an open region containing S, then

$$\int_C \mathbf{F}\cdot\mathbf{T}\ ds = \iint_S (\nabla\times\mathbf{F})\cdot\mathbf{n}\ d\sigma.$$

- Divergence Theorem: If S is a piecewise smooth closed oriented surface enclosing a volume D and $\mathbf{F}$ is a vector field whose components have continuous partial derivatives on D, then

$$\iint_S \mathbf{F}\cdot\mathbf{n}\ d\sigma = \iiint_D \nabla\cdot\mathbf{F}\ dV.$$

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