

MATH 2551 GT-E Midterm 3
VERSION B
Summer 2025
COVERS SECTIONS 15.5-15.8, 16.1-16.8

Full name: _____ **GT ID:**_____

Honor code statement: I will abide strictly by the Georgia Tech honor code at all times. I will not use a calculator. I do not have a phone within reach, and I will not reference any website, application, or other CAS-enabled service. I will not consult with my notes or anyone during this exam. I will not provide aid to anyone else during this exam.

() All of the knowledge presented in this exam is entirely my own. I am initialing to the left to attest to my integrity.

Read all instructions carefully before beginning.

- Print your name and GT ID neatly above.
- You have 75 minutes to take the exam.
- You may not use aids of any kind.
- Please show your work [J] and annotate your work using proper notation [N].
- Good luck!

Question	Points
1	2
2	2
3	2
4	8
5	6
6	10
7	8
8	10
Total:	48

For problems 1-2 choose whether each statement is true or false. If the statement is *always* true, pick true. If the statement is *ever* false, pick false. For all problems on this page please be sure to neatly fill in the bubble corresponding to your answer choice. [A]

1. (2 points) If C_1 and C_2 are two curves with the same starting point and ending point, then $\int_{C_1} \nabla f \cdot \mathbf{T} ds = \int_{C_2} \nabla f \cdot \mathbf{T} ds$.

☐ TRUE ☐ FALSE

2. (2 points) If C is a curve with parametrization $\mathbf{r}_1(t) = \langle x(t), y(t) \rangle$ with $t \in [a, b]$, then the parametrization $\mathbf{r}_2(t) = \mathbf{r}_1(-t)$ with $t \in [a, b]$ is also a parametrization of C with the opposite orientation. [A]

☐ TRUE ☐ FALSE

3. (2 points) If $g(x, y)$ is strictly positive at all points in $\mathbb{R}^2$, then

$$\iint_{R_1} g(x, y) dA = \iint_{R_2} g(x, y) dA$$

if the area of R_1 is equal to the area of R_2 . [A]

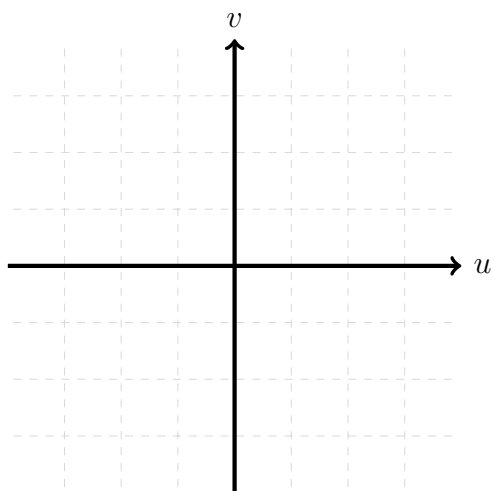
☐ TRUE ☐ FALSE

4. (8 points) In this problem you will compute the integral

$$M = \iint_R (x + y)e^{x^2 - y^2} dx dy$$

for the region R in the first quadrant bounded by the lines $x - y = 0$, $x - y = 1$, $x + y = 0$, $x + y = 2$ using the transformation $u = x - y$, $v = x + y$.

- (a) On the axes provided, sketch the new region of integration G in the uv -plane. [A]
 (b) Solve the transformation equations for x and y in terms of u and v and compute the Jacobian determinant, i.e., find $\mathbf{T}(u, v)$ and $|\det D\mathbf{T}(u, v)|$. [AJN]
 (c) Evaluate the double integral using a change of variables and your results from parts (a) and (b) to find the value of M . *Hint: integrate with the order $du dv$.* [AJN]



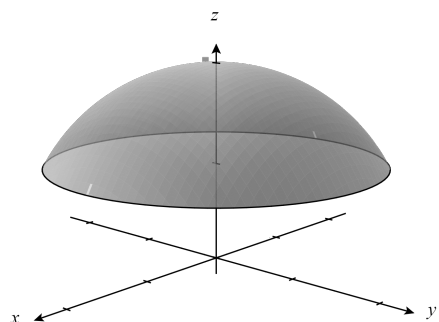
$\mathbf{T}(u, v) =$

$|\det D\mathbf{T}(u, v)| =$

5. (6 points) Let D be the smaller cap cut from a solid ball of radius 1 units by the plane $z = \sqrt{3}/2$. The point $P(-\frac{1}{2}, 0, \frac{\sqrt{3}}{2})$ is on D at the intersection of the ball and the plane.

(a) Find the spherical coordinates (ρ, φ, θ) of the point $P(-\frac{1}{2}, 0, \frac{\sqrt{3}}{2})$. [AN]

(b) Express the volume of D as an iterated triple integral in spherical coordinates. *Do not evaluate!* [AN]



Volume=

$$(-\frac{1}{2}, 0, \frac{\sqrt{3}}{2})_{\mathcal{S}} =$$

6. (10 points) This problem will have you compute the flux of the vector field $\mathbf{F}(x, y, z) = x\mathbf{i} + y\mathbf{j}$ through S the open ended circular cylinder of radius 5 and height 3 with its base on the xy -plane and centered about the z -axis, oriented away from the z -axis. [AJN]
- (a) Set up and evaluate a surface integral which computes the flux of $\mathbf{F}$ through S .
- (b) Why can we NOT evaluate a line integral over the boundary C to obtain the result of part (a) using Stokes' Theorem? Use at least one or two complete sentences to answer.

7. (8 points) Consider the vector field $\mathbf{F} = \langle zx, zy, z \rangle$ and the surface S consisting of the portion of the paraboloid $z = 4 - x^2 - y^2$ with $z \geq 0$ together with the circular disk $x^2 + y^2 \leq 4$ in the xy -plane, oriented with normal vectors away from the origin. [AJN]
- (a) Use the Divergence Theorem to set up a triple iterated integral over the region D which is enclosed by S , which evaluates to the same value as $\iint_S \mathbf{F} \cdot \mathbf{n} \, d\sigma$.
Do not evaluate!
- (b) Is there a vector field $\mathbf{G}$ such that $\mathbf{F} = \nabla \times \mathbf{G}$? Explain in one or two complete sentences how you know.

8. (10 points) Consider the vector field $\mathbf{F} = \langle x, y \rangle$ and the curve C which is the circle $x^2 + y^2 = 25$ oriented counterclockwise with outward pointing normal vector. [AJN]

- (a) Compute the flow of F around T , which is $\int_C F \cdot \mathbf{T} ds$, **using** Green's Theorem.
 - (b) Compute the flux of F around T , which is $\int_C F \cdot \mathbf{n} ds$, **with or without** using Green's Theorem. *Use either method, but you can use the other integral to check your answer.*
 - (c) Is F conservative? Explain in at least one or two complete sentences how you know.
- If you need more room, please use the next page and leave a note at the bottom of this page.*

If you need more room on the previous problem please use this page and indicate it at the bottom of the previous page.

FORMULA SHEET

- Trig identities: $\sin^2(x) = \frac{1}{2}(1 - \cos(2x))$, $\cos^2(x) = \frac{1}{2}(1 + \cos(2x))$
- Volume(D) = $\iiint_D dV$, $f_{avg} = \frac{\iiint_D f(x, y, z) dV}{\text{volume of } D}$ or $\frac{\int_C f(x, y, z) ds}{\text{length of } C}$,
Mass: $M = \iiint_D \delta dV$
- Cylindrical coordinates: $x = r \cos(\theta)$, $y = r \sin(\theta)$, $z = z$, $dV = r dz dr d\theta$
- Spherical coordinates: $x = \rho \sin(\phi) \cos(\theta)$, $y = \rho \sin(\phi) \sin(\theta)$, $z = \rho \cos(\phi)$,
 $dV = \rho^2 \sin(\phi) d\rho d\phi d\theta$
- Substitution for double integrals: If R is the image of G under a coordinate transformation $\mathbf{T}(u, v) = \langle x(u, v), y(u, v) \rangle$ then

$$\iint_R f(x, y) dx dy = \iint_G f(\mathbf{T}(u, v)) |\det D\mathbf{T}(u, v)| du dv.$$

- Scalar line integral: $\int_C f(x, y, z) ds = \int_a^b f(\mathbf{r}(t)) |\mathbf{r}'(t)| dt$
- Tangential vector line integral: $\int_C \mathbf{F} \cdot \mathbf{T} ds = \int_C \mathbf{F} \cdot d\mathbf{r} = \int_a^b \mathbf{F}(\mathbf{r}(t)) \cdot \mathbf{r}'(t) dt$
- Normal vector line integral: $\int_C \mathbf{F}(x, y) \cdot \mathbf{n} ds = \int_C P dy - Q dx = \int_a^b \mathbf{F}(\mathbf{r}(t)) \cdot \langle y'(t), -x'(t) \rangle dt$.
- Fundamental Theorem of Line Integrals: $\int_C \nabla f \cdot d\mathbf{r} = f(B) - f(A)$ if C is a path from A to B
- Curl (Mixed Partials) Test: $\mathbf{F} = \nabla f$ if $\text{curl } \mathbf{F} = \mathbf{0} \Leftrightarrow P_z = R_x, Q_z = R_y$, and $Q_x = P_y$.
- $\nabla = \left\langle \frac{\partial}{\partial x}, \frac{\partial}{\partial y}, \frac{\partial}{\partial z} \right\rangle$ $\text{div } \mathbf{F} = \nabla \cdot \mathbf{F}$ $\text{curl } \mathbf{F} = \nabla \times \mathbf{F}$
- Green's Theorem: If C is a simple closed curve with positive orientation and R is the simply-connected region it encloses, then

$$\int_C \mathbf{F} \cdot \mathbf{T} ds = \iint_R (\nabla \times \mathbf{F}) \cdot \mathbf{k} dA \quad \int_C \mathbf{F} \cdot \mathbf{n} ds = \iint_R (\nabla \cdot \mathbf{F}) dA.$$

- Surface Area = $\iint_S 1 d\sigma$
- Scalar surface integral: $\iint_S f(x, y, z) d\sigma = \iint_R f(\mathbf{r}(u, v)) |\mathbf{r}_u \times \mathbf{r}_v| dA$
- Flux surface integral: $\iint_S \mathbf{F} \cdot \mathbf{n} d\sigma = \iint_S \mathbf{F} \cdot d\boldsymbol{\sigma} = \iint_R \mathbf{F}(\mathbf{r}(u, v)) \cdot (\mathbf{r}_u \times \mathbf{r}_v) dA$
- Stokes' Theorem: If S is a piecewise smooth oriented surface bounded by a piecewise smooth curve C and $\mathbf{F}$ is a vector field whose components have continuous partial derivatives on an open region containing S , then

$$\int_C \mathbf{F} \cdot \mathbf{T} ds = \iint_S (\nabla \times \mathbf{F}) \cdot \mathbf{n} d\sigma.$$

- Divergence Theorem: If S is a piecewise smooth closed oriented surface enclosing a volume D and $\mathbf{F}$ is a vector field whose components have continuous partial derivatives on D , then

$$\iint_S \mathbf{F} \cdot \mathbf{n} d\sigma = \iiint_D \nabla \cdot \mathbf{F} dV.$$

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