§16.7 Stokes' Theorem

Theorem 157 (Stokes' Theorem)**.** *Let S be a smooth oriented surface and C be its compatibly oriented boundary. Let $\mathbf{F}$ be a vector field with continuous partial derivatives. Then*

$$\iint_S (\nabla \times \mathbf{F}) \cdot \mathbf{n} \, d\sigma = \int_C \mathbf{F} \cdot \mathbf{T} \, ds.$$

Example 158 (DD)**.** Let $\mathbf{F} = \langle -y, x+(z-1)x^{x\sin(x)}, x^2+y^2\rangle$. Find $\iint_S (\nabla\times\mathbf{F})\cdot\mathbf{n}\,d\sigma$ over the surface S which is the part of the sphere $x^2+y^2+z^2=2$ above $z=1$, oriented away from the origin.

Question: What can we say if two different surfaces S_1 and S_2 have the same oriented boundary C?

Example 159. Suppose $\operatorname{curl} \mathbf{F} = \langle y^{y^y} \sin(z^2), (y-1)e^{x^{x^x}} + 2, -ze^{x^{x^x}} \rangle$. Compute the net flux of the curl of $\mathbf{F}$ over the surface pictured below, which is oriented outward and whose boundary curve is a unit circle centered on the y-axis in the plane $y = 1$.

§16.8 Divergence Theorem

Theorem 160 (Divergence Theorem)**.** *Let S be a closed surface oriented outward, D be the volume inside S, and $\mathbf{F}$ be a vector field with continuous partial derivatives. Then*

$$\iint_S \mathbf{F} \cdot \mathbf{n}\, d\sigma = \iiint_D \nabla \cdot \mathbf{F}\, dV.$$

Example 161. Let $\mathbf{F} = \langle y^{1234} e^{\sin(yz)}, y - x^{z^x}, z^2 - z \rangle$ and S be the surface consisting of the portion of cylinder of radius 1 centered on the z-axis between $z = 0$ and $z = 3$, together with top and bottom disks, oriented outward. Find the flux of $\mathbf{F}$ through S.