

Full name: Key GT ID: _____ Sec: _____

Quiz 10 Version A

You have 15 minutes to take the quiz. No phones, notes, or use aids of any kind is permitted.

1. (4 points) [Parameterizations of Curves and Line Integrals] True or False.

(a) The vector field $\mathbf{F} = \langle y + z, z, x + y \rangle$ is conservative.☐ TRUE☒ FALSE

THM: (Curl test)

 $\mathbf{F} = \nabla f$ for some $f \iff \text{Curl } \mathbf{F} = \vec{0}$. [A]

$$\text{Curl } \mathbf{F} = \nabla \times \mathbf{F} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ y+z & z & x+y \end{vmatrix} = \langle 1-1, -(1-1), 0-1 \rangle = \langle 0, 0, -1 \rangle \neq \langle 0, 0, 0 \rangle$$

(b) Find a potential function for $\mathbf{F} = \langle y \sin z, x \sin z, xy \cos z \rangle$.

[AN]

$$f = \int f_x dx = xy \sin z + C(y, z)$$

$$\text{So } f_y = x \sin z + C_y(y, z) = x \sin z \Rightarrow C_y(y, z) = C_z(z)$$

$$f_z = xy \cos z + C'(z) = xy \cos z \Rightarrow C(z) = C$$

$$f(x, y, z) = xy \sin z$$

2. (6 points) [Line Integrals of Scalar Functions]

Evaluate the line integral $\int_C \mathbf{F} \cdot \mathbf{T} \, ds$ using FToLI.

[AJN]

$$\mathbf{F} = \left\langle 3x^2, \frac{z^2}{y}, 2z \ln y \right\rangle,$$

$$C : \mathbf{r}(t) = (1-t)\langle 1, 1, 1 \rangle + t\langle 1, 2, 3 \rangle, \quad t \in [0, 1].$$

Find potential function for $\mathbf{F}$

$$\int f_x dx = \int 3x^2 dx = x^3 + C(y, z)$$

$$\int f_y dy = \int \frac{z^2}{y} dy = z^2 \ln y + C(z)$$

$$\text{So } f_z = z \ln y + C'(z) = z \ln y \Rightarrow C(z) = C.$$

$$\text{So } \mathbf{F} = \nabla f$$

where

$$f = x^3 + z^2 \ln y$$

$$\text{Next } \mathbf{r}(0) = \langle 1, 1, 1 \rangle \text{ and } \mathbf{r}(1) = \langle 1, 2, 3 \rangle$$

By FToLI

$$\int_C \mathbf{F} \cdot \mathbf{T} \, ds = f(1, 2, 3) - f(1, 1, 1)$$

$$= (1^3 + 3^2 \ln 2) - (1^3 + 1^2 \ln 1) = 9 \ln 2$$