MATH 2551-GT-E– Multivariable Calculus Summer 2025

Full name: Key **GT ID:**________ **Sec:**____

Quiz 10 Version B

You have 15 minutes to take the quiz. No phones, notes, or use aids of any kind is permitted.

1. (4 points) **[Parameterizations of Curves and Line Integrals]** *True or False.*

   (a) The vector field $\mathbf{F} = \langle y + z, \overset{x+z}{\cancel{z}}, x + y\rangle$ is conservative. [A]

   THM: (Curl test)
   $\mathbf{F} = \nabla f$ for some $f \Leftrightarrow \operatorname{curl}\mathbf{F} = \vec{0}$.

   - [x] TRUE
   - [ ] FALSE

   $$\operatorname{curl}\mathbf{F} = \nabla \times \mathbf{F} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ y+z & x+z & x+y \end{vmatrix} = \langle 1-1, -(1-1), 1-1\rangle = \langle 0,0,0\rangle = \langle 0,0,0\rangle$$

   (b) Find a potential function for $\mathbf{F} = \langle 2x, -y^2, \frac{-4}{1+z^2}\rangle$. [AN]

   $$\int f_x\,dx = \int 2x\,dx = x^2 + C(y,z)$$

   So $$\int f_y\,dy = \int -y^2\,dy = -\frac{1}{3}y^3 + C(z)$$

   and $$\int f_z\,dy = \int \frac{-4}{1+z^2}\,dz = -4\tan^{-1}(z) + C$$

   $$\boxed{f(x,y,z) = x^2 - \frac{1}{3}y^3 - 4\tan^{-1}(z)}$$

2. (6 points) **[Line Integrals of Scalar Functions]**

   Evaluate the line integral $\int_C F \cdot T\,ds$ using FToLI. [AJN]

   $$\mathbf{F} = \left\langle \frac{1}{y}, \frac{1}{z} - \frac{x}{y^2}, \frac{-y}{z^2} \right\rangle,$$

   $$C : \mathbf{r}(t) = \langle 1, 1, 1\rangle + t\langle 1, 1, 3\rangle,\ t \in [0, 1].$$

   Find potential function for F

   $$\int f_x\,dx = \int \frac{1}{y}\,dy = \frac{x}{y} + C(y,z)$$

   So $$f_y = \frac{-x}{y^2} + C_y(y,z) = \frac{-x}{y^2} + \frac{1}{z} \Rightarrow C_y(y,z) = \frac{1}{z} + C(z).$$

   Then $$\int f_y\,dy = \int \frac{-x}{y^2} + \frac{1}{z} + C(z)\,dy = \frac{x}{y} + \frac{y}{z} + C(z)y$$

   and $$f_z = 0 - \frac{y}{z^2} + y\,C'(z) = \frac{-y}{z^2} \Rightarrow C'(z) = 0 \Rightarrow C(z) = C.$$

   So $f(x,y,z) = \frac{x}{y} + \frac{y}{z}$ is a potential function for F.

   Now $r(0) = \langle 1,1,1\rangle$ and $r(1) = \langle 2,2,4\rangle$.

   So by FToLI

   $$\int_C F \cdot T\,ds = f(2,2,4) - f(1,1,1)$$
   $$= \left(\frac{2}{2} + \frac{2}{4}\right) - \left(\frac{1}{1} + \frac{1}{1}\right) = 1 + \frac{1}{2} - 2 = \boxed{-\frac{1}{2}}$$