MATH 2551-GT-E– Multivariable Calculus **Summer 2025**

Full name: Key **GT ID:** **Sec:**

Quiz 2 Version A

You have 15 minutes to take the quiz. No phones, notes, or use aids of any kind is permitted.

1. (2 points) Choose whether the following statement is true or false. [A]

   The surface defined by $x^2 = y^2 + z^2$ has circular cross sections when intersected by a plane parallel to the yz-coordinate plane, except for the cross section which contains the origin.

   ● **TRUE** ○ **FALSE**

   plane parallel to yz-coord plane given by eqn. $x = k$ (constant), so eqn ★ becomes $k^2 = y^2 + z^2$ is a circle.

2. (8 points) **[Arc length]**

   Consider the curve parametrized by $\mathbf{r}(t) = \langle 2\sin t, 2\cos t, t\rangle,\ 0 \le t \le \pi$.

   (a) Find the arc length parameter $s(t)$ along the curve from the point where $t = 0$ by evaluating the integral [AJN]

   $$s(t) = \int_0^t |\mathbf{v}(\tau)|\, d\tau$$

   (b) Use your answer to part (a) to find L the length of the indicated portion of the curve. [AJN]

(a) $\vec{v}(t) = \vec{r}'(t) = \langle 2\cos t, -2\sin t, 1\rangle$

$|\vec{v}(t)| = \sqrt{4\cos^2 t + 4\sin^2 t + 1} = \sqrt{5}$ (the $4\cos^2 t + 4\sin^2 t$ part equals 4)

$s(t) = \int_0^t |\vec{v}(\tau)|\, d\tau = \int_0^t \sqrt{5}\, d\tau = \sqrt{5}\,\tau \Big|_0^t$

$= \boxed{\sqrt{5}\,t}$

(b) $L = \int_0^{\pi} |\vec{v}(t)|\, dt = s(\pi) = \boxed{\sqrt{5}\,\pi}$