Full name: $\quad$ Key $\qquad$ **GT ID:** $\qquad$ **Sec:** $\qquad$

Quiz 2 Version B

You have 15 minutes to take the quiz. No phones, notes, or use aids of any kind is permitted.

1. (2 points) Choose whether the following statement is true or false. [A]

   The parameterization $\mathbf{r}(s) = \langle 2\cos\left(\frac{s}{3}\right), 2\sin\left(\frac{s}{3}\right), \frac{s}{3}\rangle$, $0 \le s \le \pi$, is an arc-length parameterization of it's graph.

   ○ **TRUE** $\qquad$ ● **FALSE**

   $$r'(t) = \langle -\tfrac{2}{3}\sin\left(\tfrac{s}{3}\right), \tfrac{2}{3}\cos\left(\tfrac{s}{3}\right), \tfrac{1}{3}\rangle$$

   $$|s'(t)| = \sqrt{\tfrac{4}{9}\sin^2\left(\tfrac{s}{3}\right) + \tfrac{4}{9}\cos^2\left(\tfrac{s}{3}\right) + \tfrac{1}{9}} = \sqrt{5/9} \ne 1.$$

   $\underbrace{\qquad\qquad}_{4/9}$

2. (8 points) [**Unit Tangent & Normal, Curvature**]

   Consider the curve parameterized by $\mathbf{r}(t) = \langle 3\cos t, 3\sin t, 4t\rangle$, $0 \le t \le \pi$. Find $\mathbf{T}$, $\mathbf{N}$, and κ for the space curve defined by $\mathbf{r}$. [AJN]

$$T(t) = \frac{\vec{r}'(t)}{|\vec{r}(t)|} \qquad \vec{r}'(t) = \langle -3\sin t, 3\cos t, 4\rangle$$

$$|\vec{r}'(t)| = \sqrt{\underbrace{9\sin^2 t + 9\cos^2 t}_{9} + 16} = \sqrt{9+16} = \sqrt{25} = 5$$

So $\boxed{T(t) = \langle -\tfrac{3}{5}\sin t, \tfrac{3}{5}\cos t, \tfrac{4}{5}\rangle}$

$$N(t) = \frac{T'(t)}{|T'(t)|} \qquad T'(t) = \langle -\tfrac{3}{5}\cos t, -\tfrac{3}{5}\sin t, 0\rangle$$

$$|T'(t)| = \sqrt{\tfrac{9}{25}\cos^2 t + \tfrac{9}{25}\sin^2 t} = \sqrt{\tfrac{9}{25}} = \tfrac{3}{5}$$

So $N(t) = \frac{1}{3/5}\langle -\tfrac{3}{5}\cos t, -\tfrac{3}{5}\sin t, 0\rangle$

$$= \tfrac{5}{3}\langle -\tfrac{3}{5}\cos t, -\tfrac{3}{5}\sin t, 0\rangle$$

$$\boxed{N(t) = \langle -\cos t, -\sin t, 0\rangle}$$

$$\kappa = \frac{|T'(t)|}{|r'(t)|} = \frac{3/5}{5} = \tfrac{3}{5} \times \tfrac{1}{5} = \boxed{3/25}$$