MATH 2551-GT-E– Multivariable Calculus **Summer 2025**

Full name: Key **GT ID:**____________ **Sec:**____

Quiz 4 Version B

You have 15 minutes to take the quiz. No phones, notes, or use aids of any kind is permitted.

1. (3 points) If $F(x, y, z) = c$ is an equation used to implicitly define z as a function of x and y, then: *Fill in the blanks for the missing formula* [AN]

$$\frac{\partial z}{\partial x} = \boxed{\frac{-F_x}{F_z}} \qquad \frac{\partial z}{\partial y} = \boxed{\frac{-F_y}{F_z}}$$

2. (7 points) **[Chain Rule]**
Suppose that $W(s, t) = F(\underbrace{\mathbf{u}(s, t), \mathbf{v}(s, t)}_{g(u,v)})$, where $\mathbf{u}, \mathbf{v}, F$ are differentiable functions and we know the following information.

$$\begin{aligned} u(2, 1) &= 4 & v(2, 1) &= 5 \\ u_s(1, 0) &= 3 & v_s(1, 0) &= -2 \\ u_t(1, 0) &= -1 & v_t(1, 0) &= 4 \\ F_u(4, 5) &= 7 & F_v(4, 5) &= -6 \end{aligned}$$

First express DW, the total derivative of W, symbolically as the product of the two matrices DF and Dg. Then, evaluate $DW|_{(s,t)=(2,1)}$ and identify $W_s(2, 1)$ and $W_t(2, 1)$. [AJN]

$$W(s,t) = F(g(s,t)), \quad g(s,t) = \langle u(s,t), v(s,t) \rangle$$

$$\underset{1\times 2}{DW} = DF * Dg = \underset{1\times 2}{\begin{bmatrix} F_u & F_v \end{bmatrix}} * \underset{2\times 2}{\begin{bmatrix} u_s & u_t \\ v_s & v_t \end{bmatrix}}$$

@ $(s,t) = (2,1)$
$(u,v) = (4,5)$

$$DW|_{@(2,1)} = \begin{bmatrix} 7 & -6 \end{bmatrix} \begin{bmatrix} 3 & -1 \\ -2 & 4 \end{bmatrix} = \begin{bmatrix} 33 & -31 \end{bmatrix}$$

So & $\boxed{\begin{aligned} W_s(2,1) &= 33 \\ W_t(2,1) &= -31 \end{aligned}}$