MATH 2551-GT-E– Multivariable Calculus **Summer 2025**

Full name: Key **GT ID:**________ **Sec:**___

Quiz 5 Version A

You have 15 minutes to take the quiz. No phones, notes, or use aids of any kind is permitted.

1. (4 points) **[Gradient]**
   Find a unit vector in the direction of maximum increase and a unit vector in the direction of maximum decrease for the function $f(x, y) = xe^y + z^2$ at $P(1, \ln 2, \frac{1}{2})$. [AJN]

Idea: $\nabla f(P)$ points in direction of steepest ascent!

$\nabla f = \langle e^y, xe^y, 2z \rangle$, so $\nabla f(P) = \langle e^{\ln 2}, 1e^{\ln 2}, 1 \rangle$

$= \langle 2, 2, 1 \rangle$ length is $\sqrt{9} = 3$

So $\boxed{\begin{array}{l} \text{max increase } \vec{u} = \langle 2/3, 2/3, 1/3 \rangle \\ \text{max decrease } -\vec{u} = \langle -2/3, -2/3, -1/3 \rangle \end{array}}$

2. (8 points) **[Tangent Planes and Normal Lines]**
   Find the tangent plane of the surface and the equation of the normal line at the given point. [AJN]

$$z = (x + y + 2)^2, \text{ at } Q(1, 2, 25).$$

Equation $z = z_0 + f_x(P)(x - x_0) + f_y(P)(y - y_0)$

$\nabla f = \langle f_x \; f_y \rangle = \langle 2(x+y+2), 2(x+y+2) \rangle$

$\nabla f(P) = \langle 10, 10 \rangle$ so tangent plane is

$$\boxed{z = 25 + 10(x-1) + 10(y-2)}$$

and normal line is $l(t) = \vec{P} + t\vec{v}$ where $\vec{v}$ normal to plane

$$\boxed{l(t) = \langle 1, 2, 25 \rangle + t\langle 10, 10, -1 \rangle}$$