

Full name: _____ GT ID: _____ Sec: _____

Key

Quiz 6 Version A

You have 15 minutes to take the quiz. No phones, notes, or use aids of any kind is permitted.

1. (10 points) [Lagrange Multipliers]

Find the maximum and minimum value of $f(x, y)$ subject to the constraint $g(x, y) = k$ using the method of Lagrange multipliers. [AJN]

$$f(x, y) = x^2y, \text{ subject to } x + y = 3.$$

$$f(x, y) = x^2y, \text{ subject to } g(x, y) = x + y = 3.$$

$$\text{Set up } \nabla f = \langle 2xy, x^2 \rangle \quad \nabla g = \langle 1, 1 \rangle$$

$$\text{Solve } \begin{cases} \textcircled{1} 2xy = \lambda \\ \textcircled{2} x^2 = \lambda \\ \textcircled{3} x + y = 3 \end{cases}$$

Use $\textcircled{1}$ & $\textcircled{2} \Rightarrow \lambda = \lambda$ to get

$$2xy = x^2 \Rightarrow x^2 - 2xy = 0 \Rightarrow x(x - 2y) = 0$$

CASE I. CASE II.
So $x = 0$ or $x = 2y$.

In Case I: $x = 0, y = 3, \lambda = 0$ get $(0, 3)$

In Case II: $\begin{cases} \textcircled{1} 4y^2 = \lambda \\ \textcircled{2} 3y = 3 \end{cases} \Rightarrow y = 1 \text{ \& } \lambda = 4.$
get $(2, 1)$.

So MAX of f is 4 at $(2, 1)$
MIN of f is 0 at $(0, 3)$

Evaluate

(x, y)	$f(x, y)$
$(0, 3)$	0 \leftarrow MIN
$(2, 1)$	4 \leftarrow MAX