MATH 2551-GT-E– Multivariable Calculus **Summer 2025**

Full name: Key **GT ID:** _____ **Sec:** ____

Quiz 6 Version B

You have 15 minutes to take the quiz. No phones, notes, or use aids of any kind is permitted.

1. (5 points) **[Critical Points]**
Find all local maxima, local minima, and saddle points of the function $f(x, y)$. [AJN]

$$f(x, y) = x^4 + y^4 + 4xy.$$

Step 1: $Df = [4x^3+4y \quad 4y^3+4x] = [0 \ 0]$

Step 2: Solve $\begin{cases} ① \ 4x^3+4y=0 \\ ② \ 4y^3+4x=0 \end{cases} \Rightarrow \begin{cases} ① \ -x^3=y \\ ② \ -y^3=x \end{cases}$

Sub ① into ② to get

$-(-x^3)^3 = x \rightarrow -x^6 = x \Rightarrow x^6+x=0$

$\Rightarrow x(x^5+1)=0 \Rightarrow x=0$ or $x^5=-1$

$\Rightarrow x=0$ or $x=-1$

Now sub $x=0$ or $x=-1$ back into ① to get

when $x=0$, $y=0$

when $x=-1$, $y=1$

So Two Solutions are $\boxed{(0,0) \text{ & } (-1,1)}$

2. (5 points) **[Lagrange Multipliers]**
Find the maximum and minimum value of $f(x, y)$ subject to the constraint $g(x, y) = k$ using the method of Lagrange multipliers. [AJN]

$$f(x, y) = x^2 + y^2 + z^2, \text{ subject to } x + y + z = 9.$$

Set up: $\nabla f = \langle 2x, 2y, 2z \rangle$ and $\nabla g = \langle 1, 1, 1 \rangle$

Solve $\begin{cases} 2x = \lambda \\ 2y = \lambda \\ 2z = \lambda \\ x+y+z=9 \end{cases}$ So $3\left(\frac{\lambda}{2}\right)=9 \Rightarrow \lambda=6$, $\boxed{x=y=z=3.}$