

Full name: _____ GT ID: _____ Sec: _____

Quiz 8 Version A

You have 15 minutes to take the quiz. No phones, notes, or use aids of any kind is permitted.

1. (4 points) [Cylindrical and Spherical Coordinates] Fill in the blanks. [AN]

(a) Find the spherical coordinates of the point $(x, y, z) = (0, 3\sqrt{2}, 3\sqrt{2})$.

$$\rho = \sqrt{0^2 + 18 + 18} = \sqrt{36} = 6 \quad \theta = \pi/2, \quad \phi = \pi/4$$

$$(6, \pi/4, \pi/2)$$

(b) State the volume differential dV for spherical coordinates.

$$dV = \rho^2 \sin \phi \, d\rho \, d\phi \, d\theta$$

2. (6 points) [Triple Integrals in Rectangular Coordinates]

Set up and evaluate a triple iterated integral in cartesian (rectangular) coordinates which computes the volume of the region R . The region R is the tetrahedron in the first octant bounded by the coordinate planes and the plane $x + y + z = 3$.Hint: $dV = dz \, dy \, dx$

[AJN]

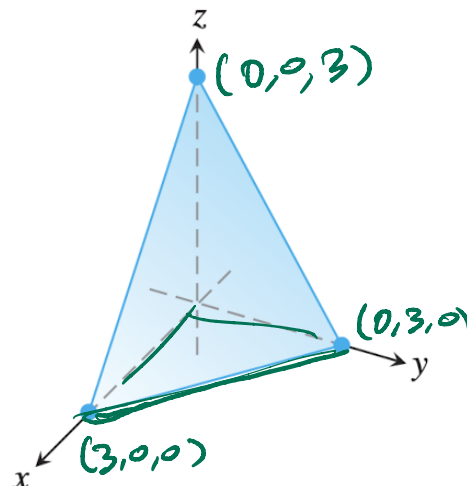
$$z = 3 - x - y$$

$$x \in [0, 3]$$

$$y \in [0, 3 - x]$$

$$z \in [0, 3 - x - y]$$

$$V = \int_0^3 \int_0^{3-x} \int_0^{3-x-y} 1 \, dz \, dy \, dx$$



$$= \int_0^3 \int_0^{3-x} (3-x-y) \, dy \, dx = \int_0^3 (3y - xy - \frac{1}{2}y^2) \, dy \, dx$$

$$= \int_0^3 (3-x)(3-x) - \frac{1}{2}(3-x)^2 \, dx = \int_0^3 \frac{1}{2}(3-x)^2 \, dx$$

$$\begin{aligned} u &= 3-x \\ du &= -dx \\ x=0 &\Rightarrow u=3 \\ x=3 &\Rightarrow u=0 \end{aligned}$$

$$= \frac{1}{2} \int_3^0 u^2 \, du = -\frac{1}{2} \frac{1}{3} u^3 \Big|_3^0 = -\frac{1}{6} (0 - 3^3) = \frac{9}{2}$$