MATH 2551-GT-E– Multivariable Calculus **Summer 2025**

Full name: Key **GT ID:** _____ **Sec:** ____

Quiz 8 Version A

You have 15 minutes to take the quiz. No phones, notes, or use aids of any kind is permitted.

1. (4 points) **[Cylindrical and Spherical Coordinates]** *Fill in the blanks.* [AN]

   (a) Find the spherical coordinates of the point $(x, y, z) = (0, 3, 3\sqrt{3})$.

   $\rho = \sqrt{9+27} = \sqrt{36} = 6$, $\theta = \frac{\pi}{2}$, $\alpha = \pi/3$ s. $\varphi = \pi/6$

   $\boxed{(6, \pi/6, \pi/2)}$

   (b) State the volume differential dV for spherical coordinates.

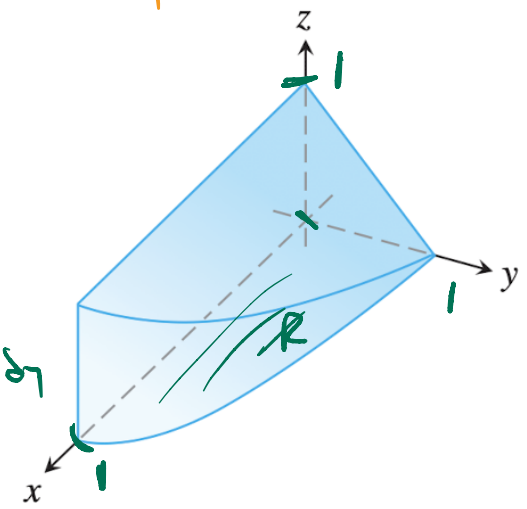
   $dV = \boxed{\rho^2 \sin\varphi \, d\rho \, d\varphi \, d\theta}$

2. (6 points) **[Triple Integrals in Rectangular Coordinates]**
   Set up and evaluate a triple iterated integral in cartesian (rectangular) coordinates which computes the volume of the region D. The region D is the region bounded by the coordinate planes, the plane $y + z = 1$, and the cylinder $x = 1 - y^2$. *Hint:* $dV = dz\,dx\,dy$ [AJN]

$y \in [0, 1]$

$x \in [0, 1-y^2)$

$z \in [0, 1-y)$



$$\text{So } \text{Vol} = \int_0^1 \int_0^{1-y^2} \int_0^{1-y} 1 \, dz\,dx\,dy$$

$$= \int_0^1 \int_0^{1-y^2} z \Big|_0^{1-y} dx\,dy$$

$$= \int_0^1 \int_0^{1-y^2} 1-y \, dx\,dy = \int_0^1 (1-y)x \Big|_0^{1-y^2} dy$$

$$= \int_0^1 (1-y)(1-y^2)\,dy = \int_0^1 1 - y - y^2 + y^3 \, dy$$

$$= y - \frac{1}{2}y - \frac{1}{3}y^2 + \frac{1}{4}y^3 \Big|_0^1 = 1 - \frac{1}{2} - \frac{1}{3} + \frac{1}{4} = \frac{12-6-4+3}{12} = \boxed{5/12}$$