Practice Exam 1

1. Valid? Prove or disprove.

(a)
$$\begin{array}{l} p \to q \\ \underline{q \vee r} \\ r \to (\neg q) \end{array}$$

(b)
$$\begin{array}{c} p \to q \\ (\neg r) \vee (\neg q) \\ \underline{r} \\ (\neg p) \end{array}$$

2. Valid? Prove or disprove.

If I work hard, then I earn lots of money.
<u>If I don't pay high taxes, then I don't work hard.</u>
If I work hard, then I pay high taxes.

3. True or False questions.

(i) If $p \wedge q$ is true, then $p \vee q$ is true.

(ii) If $p \to q$ is true and $q \to p$ is true, then p is logically equivalent to q.

(iii) If $\mathcal{A}$ is a tautology and $\mathcal{B}$ is a contradiction, then $\mathcal{A} \wedge (\neg \mathcal{B})$ is a tautology.

(iv) If $\mathcal{A} \iff \mathcal{B}$ and $\mathcal{C}$ is any statement, then $(\mathcal{A} \to \mathcal{C}) \iff (\mathcal{B} \to \mathcal{C})$.

(v) If the premises of an argument are all contradictions, then the argument is valid.

(vi) If there exists a premise of an argument that is a contradiction, then the argument is valid.

(vii) If an argument is valid, then some premise of the argument is a contradiction.

(viii) The statement $(p \to q) \leftrightarrow (q \wedge (r \to s)$ evaluates to TRUE when all the atomic statements p, q, r, s are true.

(ix) If $f : A \to B$ is a function and $|A| < |B|$ then f is one-to-one.

4. Let $A = \{1, 2, 3\}$ and $B = \{2, 4, 6, 8\}$. Find $A \cap B$, $A \cup B$ and $(A \cap B) \times (A \cup B)$.

5. Let A and B be sets. Prove or give a counterexample: If $A \subseteq B$ and $A \subseteq B^c$, then $A = \emptyset$.

6. Let A, B, and C be sets. Prove or give a counterexample:

$$\text{If } C \subseteq (A \cup B), \text{ then } C \subseteq A \text{ or } C \subseteq B.$$

7. Let $f : A \to B$ and $g : B \to C$ be functions. Prove that if g is onto and f is onto then $g \circ f$ is onto. Is the converse true? Is the statement still true if we relax the assumption to only insist that g was onto? that f was onto?