Algorithms worksheet

Work in groups or alone and use the time today to answer all of the questions below.
You may use your books or notes from class, and do **not** turn in this assignment: it is yours to study for the exam next week. Recall that:

Defn: An *algorithm* consists of three stages: input, procedure, and output. All **arithmetic operations** $(+, -, \times, \div)$ and **comparisons** $(\leq, <, =)$ must occur in the procedure stage. The complexity of the algorithm is the total number of arithmetic operations and/or comparisons used for the algorithm to run for an input in the worst case.

Defn: A recurrence relation $a_n = ra_{n-1} + sa_{n-2}$ has *solution*

$$a_n = \begin{cases} c_1 x_1^n + c_2 x_2^n & \text{if } x_1 \neq x_2, \\ c_1 x_0^n + c_2 n x_0^n & \text{if } x_1 = x_2 = 0, \end{cases}$$

where x_1, x_2 are the roots of the characteristic polynomial

$$x^2 - rx - s = (x - x_1)(x - x_2) = 0,$$

and c_1, c_2 are unknown constants which are solved for using the initial conditions of the recursively defined sequence.

Defn: A *list* of elements of a set A is an ordered n-tuple, or element of $A^n = \underbrace{A \times A \times \cdots \times A}_{n \text{ times}}$.

1. Find a recursive formula and a closed formula for the sequence -1, 17, 11, 113, 179, 857 ...

2. Write an algorithm that outputs the nth term of the sequence above using the recursive definition of the sequence. Now write an algorithm that outputs the nth term of the sequence using the closed formula using only arithmetic operations. You may **not** use "exponents" but only multiplication $\times$. Find the complexity of each algorithm.

3. Write an algorithm takes takes as input a list of real numbers $\mathcal{L} = (a_1, \ldots, a_n)$ and outputs *(1)* the number of times the number 5 appears in the list, and *(2)* the number of times an entry is between 0 and 5, non-inclusive.

4. Describe the relationship between the input and the output of the algorithm below. In other words, what does the algorithm do? What is the maximum value of s if $d = n = 5$? What is the complexity of the algorithm for fixed d? for fixed n?

Input: A natural number d and a list $\mathcal{L} = (a_1, \ldots, a_n)$ of real numbers.
Procedure: Initialize: s=0.
Step 1:
For $i = 1, \ldots, n$
  For $j = 1, \ldots, d$
    If $a_i = j$, then set s to $s + 1$.
Output: s.