

**Practice Exam 2 (approximately 2x length of Exam 2)**

1. Prove that

$$1 + 2 + 2^2 + \cdots + 2^n = 2^{n+1} - 1.$$

2. Prove or disprove:

$$2 + 4 + 6 + 8 + \cdots + 2n = (n - 1)(n + 2).$$

3. In the math department there are 30 personal computers (PCs).

- 20 have A drives,
- 8 have 19-inch monitors,
- 25 are running Windows XP,
- 20 have at least two of these properties,
- 6 have all three properties.

(a) How many PCs have at least one property?

(b) How many have none of these properties?

(c) How many have exactly one?

4. How many ways can you get a total of 6 when rolling two dice?

5. How many three digit numbers contain the digits 2 and 5 but not 0, 3, or 7?

6. In a group of 29 people, how many people must there be whose birthdays are in the same month?

7. Consider the recursively defined sequence defined by  $a_0 = 2$ ,  $a_1 = -1$ , and

$$a_n = -2a_{n-1} - 8a_{n-2}, \text{ for } n \geq 2.$$

(a) Find the first five terms of the sequence  $a_n$ .

(b) Solve the recurrence to find a closed form for  $a_n$ ,  $n \geq 0$ .

(c) What is the tenth term of the sequence?

8. Consider the following algorithm.

**Input:**  $a_1, \dots, a_n$  real numbers.

**Procedure:** Initialize: Set  $s = 1$  and  $t = a_1$ .

*Step 1:* For  $i = 2, \dots, n$ ,

If  $a_i < t$  set  $s = 1$  and  $t = a_i$ ,

If  $a_i = t$  set  $s$  to  $s + 1$ .

**Output:**  $s, t$ .

(a) Describe the relationship between the output of the algorithm and the input values  $a_1, \dots, a_n$ . That is, what are  $s$  and  $t$ ?

(b) Find an accurate bound on the complexity of the algorithm in terms of the number of comparisons used. *Answer each question separately.*

9. Over a total of 11 years of teaching I have had 1,500 students. Find the largest integer  $n$  such that at least  $n$  of my previous students have the same initials. Recall, a person's initials are a string of the form  $X.Y$  where  $X$  is the first letter of their first name and  $Y$  is the first letter of their last name (for example, my initials are S.B.).

10. The number of 5-letter words (not necessarily real English words) containing the string R-A-P, where the letters are consecutive and in order, is

11. A license plate is a string of 7 characters. How many license plates can be made if the license plate needs to contain 3 different letters and 4 different numbers?

12. How many license plates can be made if the license plate needs to contain 3 different letters and 4 different numbers and the letters must all be to the left of all the numbers, and furthermore the numbers must be increasing from left to right and the letters must be in alphabetical order?

13. True or false questions.

(a)  $2 + 4 + 6 + \dots + 2n = n(n + 1)$  for all integers  $n \geq 1$ . TRUE    FALSE

(b) There exists an integer  $n$  such that for every integer  $m$ , if  $m \geq n$  then  $\lceil \frac{n}{m} \rceil = 1$ . TRUE    FALSE

(c) There are exactly 8 subsets of  $\{a, b, c\}$ , including the emptyset and the set itself. TRUE    FALSE