Practice Exam 3

1. Give an example of a planar graph with 5 vertices which has chromatic number 4, and draw a 4-colored planar model for your graph. Is it always true that no matter what graph you picked, it is not bipartite? Explain.

2. Give an example of a graph with 8 vertices that is Eulerian but not Hamiltonian.

3. Consider the following graph

$$\mathcal{G} = (\{a, b, c, d, e, f\}, \{ab, ac, ad, ae, af, bc, bd, be, bf, cd, ce, de, ef\}).$$

(a) Give a model for $\mathcal{G}$.

(b) Find a Hamiltonian path in $\mathcal{G}$.

(c) Find a subgraph of $\mathcal{G}$ that is isomorphic to K_4.

(d) Find a subgraph of $\mathcal{G}$ that is homeomorphic to K_5. Is $\mathcal{G}$ planar?

(e) Is $\mathcal{G}$ Eulerian? Justify your answer.

(f) Find the degree sequence of $\mathcal{G}$.

(g) Find two non-isomorphic spanning trees of $\mathcal{G}$.

4. Give an example of two graphs with 6 vertices that have the same degree sequence but are not isomorphic.

5. Consider the following graphs.

$$\mathcal{G} = (\{a, b, c, d, e, f\}, \{ab, ac, ad, bd, cd, de, df\}),$$
$$\mathcal{H} = (\{x, y, z, w\}, \{xy, xz, xw\}).$$

(a) Enumerate (list and number) the subgraphs of $\mathcal{G}$ that are isomorphic to $\mathcal{H}$.

(b) Enumerate the subgraphs of $\mathcal{G}$ that are isomorphic to K_3.

(c) Find the subgraphs of $\mathcal{G}$ that have a connected component isomorphic to K_3.

(d) Enumerate the spanning trees of $\mathcal{G}$. Are all spanning trees of $\mathcal{G}$ isomorphic?

6. Prove that any subgraph of a bipartite graph is bipartite.

7. Let $\mathcal{G} = (V, E)$ be a graph. Prove that if m is the minimum degree of a vertex of $\mathcal{G}$ and M is the maximum degree of a vertex of $\mathcal{G}$, then

$$m \leq \frac{2|E|}{|V|} \leq M.$$

8. Prove that the two graphs below are isomorphic by exhibiting an explicit isomorphism between them (be sure to justify that your map is an isomorphism).

$$\mathcal{G} = (\{a, b, c, d\}, \{ab, ac, ad, bc, cd\}),$$
$$\mathcal{H} = (\{x, y, z, w\}, \{xy, xw, xz, yw, yz\}).$$

9. TRUE OR FALSE Any two graphs with exactly 4 vertices and 5 edges are isomorphic.
(The answer is NOT *what you might expect!)*