

JEOPARDY: REVIEW EXAM 2

- **Big-O category**

100 pts For all $n \geq 4$, we have

$$|4n| \leq |n^2|.$$

□

Question: Show that $4n = O(n^2)$.

200 pts For all $n \geq 2$, we have

$$|5n + 1| \leq 6|n|.$$

□

Question: Show that $5n + 1 = O(n)$.

300 pts Since the limit exists and is non-zero,

$$\lim_{n \rightarrow \infty} \frac{9n^2 + 4n}{n^2} = 9.$$

□

Question: Show that $9n^2 + 4n = O(n^2)$ and $n^2 = O(9n^2 + 4n)$.

400 pts Let x_0 and $c > 0$ such that $|f(x)| \leq c|x^3|$ for $x \geq x_0$.

Then, $3x^2 \cdot f(x)$ satisfies

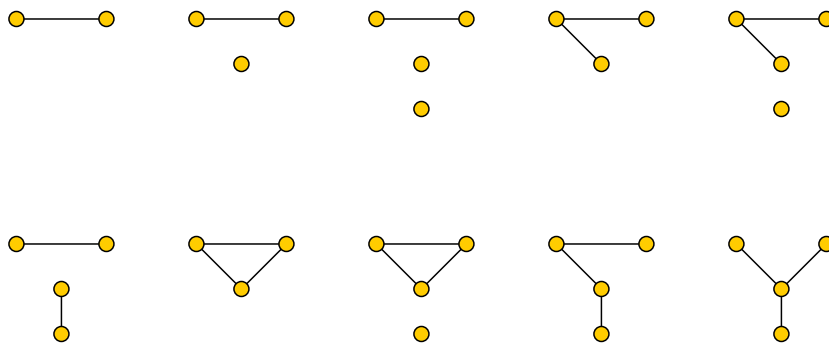
$$\begin{aligned} \forall x \geq x_0 \quad |3x^2 \cdot f(x)| &= 3x^2 \cdot |f(x)| \leq 3x^2 \cdot c|x^3| \\ &= 3c|x^5|. \end{aligned}$$

So $3x^2 f(x) = O(x^5)$.

□

Question: Prove using the definitions that if $f(x) = O(x^3)$ then $3x^2 f(x) = O(x^5)$.

• Graphs category



100 pts

□

Question: Find models for all isomorphism classes of proper subgraphs of \mathcal{G} that contain at least one edge, where

$$\mathcal{G} = (\{a, b, c, d\}, \{ab, ac, bc, cd\}).$$

Recall: a subgraph is proper if it is not empty nor the whole graph itself.

200 pts Since

$$4 + 1 + 1 + 1 + 1 = 8$$

there are $8/2 = 4$ edges.

□

Question: How many edges does a graph with degree sequence 4, 1, 1, 1, 1 have?

300 pts No, the degree sequences are not the same. The graph \mathcal{G} has degree sequence 3, 1, 1, 1, 1 and the graph \mathcal{H} has degree sequence 2, 2, 2, 0.

□

Question: Are the last two graphs with three edges from the 100pt question in this category isomorphic?

400 pts The degree sequence of \mathcal{G} is

$$3, 3, 2, 2,$$

so in particular the number of edges of \mathcal{G} is

$$|E| = \frac{3 + 3 + 2 + 2}{2} = \frac{10}{2} = 5.$$

However, \mathcal{K}_4 has $\binom{5}{2} = 6$ edges, so \mathcal{G} is not isomorphic to \mathcal{K}_4 .

□

Question: Suppose \mathcal{G} has degree sequence 3,3,2,2. Prove or disprove: \mathcal{G} is isomorphic to \mathcal{K}_4 .

• **Wrong! category**

100 pts Yes. The degree sequences are the same, they are both

$$2, 2, 2, 2, 2, 2.$$

□

Question: Are the graphs \mathcal{G} and \mathcal{H} isomorphic?

$$\begin{aligned}\mathcal{G} &= (\{a, b, c, d, e, f\}, \{ab, ac, bc, de, df, ef\}), \\ \mathcal{H} &= (\{1, 2, 3, 4, 5, 6\}, \{12, 16, 23, 34, 45, 56\}).\end{aligned}$$

200 pts Yes. The map

$$f : \{1, 2, 3\} \rightarrow \{a, b, c\}$$

defined by $f(1) = a$, $f(2) = b$, and $f(3) = c$ is bijective and each graph has exactly one edge. So the graphs are isomorphic since I exhibited an isomorphism between \mathcal{G} and \mathcal{H} .

□

Question: Prove that \mathcal{G} is isomorphic to \mathcal{H} .

$$\begin{aligned}\mathcal{G} &= (\{1, 2, 3\}, \{12\}), \\ \mathcal{H} &= (\{a, b, c\}, \{bc\}).\end{aligned}$$

300 pts Since the limit is zero, by L'Hopital's rule

$$\lim_{n \rightarrow \infty} \frac{\log_3(n)}{n^2} \stackrel{\text{L'H.}}{=} \lim_{n \rightarrow \infty} \frac{1/n}{2n} = \lim_{n \rightarrow \infty} \frac{1}{2n^2} = 0,$$

we have that

$$n^2 = O(\log_3(n)).$$

□

Question: Prove or disprove: $n^2 = O(\log_3(n))$.

400 pts If they have the same degree sequence.

If they have the same number of vertices, same number of edges, and the same degrees on all the vertices.

If they have the same number of paths all of the same lengths.

If the ways the edges line up is the same.

Also,

Because every tree is bipartite.

Because every graph which has only even cycles is bipartite.

□

Question: First part: Give a sufficient condition for two graphs to be isomorphic.

Second part: Prove that the graph

$$\mathcal{G} = (\{a, b, c, d, e\}, \{ab, ac, ad, ae\})$$

is bipartite. Prove that the graph

$$\mathcal{H} = (\{1, 2, 3, 4, 5, 6\}, \{12, 16, 13, 14, 15\})$$

is bipartite.