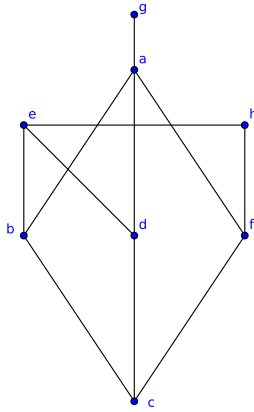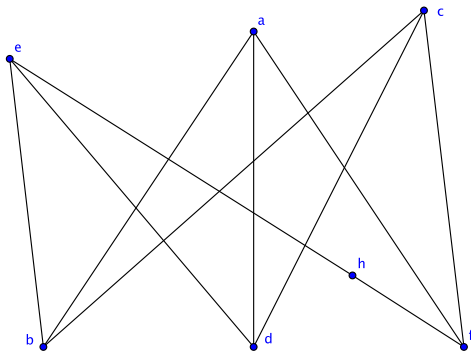# Quiz 10

For the problems below consider the following graph $\mathcal{G}$ whose model is given below:

1. Determine if the graph is planar or not. If it is planar, give a planar model. If it is not planar, find a subgraph $\mathcal{H}$ and indicate whether it is homeomorphic to K_5 or $K_{3,3}$.   (8 pts.)

Solution: The following subgraph of $\mathcal{G}$ is homeomorphic to $K_{3,3}$. It is obtained by deleting the edge (g, a) from $\mathcal{G}$.

2. Let $\mathcal{H}$ be the subgraph of $\mathcal{G}$ obtained by deleting the vertex g and the edge (g, a). Determine whether $\mathcal{H}$ is Eulerian and/or Hamiltonian. Find a Eulerian circuit and/or a Hamiltonian cycle, if possible, or state that it is not possible.   (8 pts.)

Solution: The graph is not Eulerian since not all its vertices are even. However, it is Hamiltonian. A Hamiltonian cycle is $fhebadcf$.

3. Find the chromatic number $\chi(\mathcal{G})$ and give a $\chi(\mathcal{G})$-coloring of $\mathcal{G}$.   (4 pts.)

Solution: The chromatic number is 3 (notice $\mathcal{G}$ is not bipartite even though it is homeomorphic to $K_{3,3}$ plus an edge, that is, if the vertex h were missing it *would* be bipartite with chromatic number 2!). A 3-coloring is

$$
\begin{aligned}
\text{color 1 vertices are} &\quad \{e, a, c\} \\
\text{color 2 vertices are} &\quad \{b, d, f, g\} \\
\text{color 3 vertices are} &\quad \{h\}
\end{aligned}
$$