Quiz 1

1. Find an assignment to the atomic statements p, q, r such that the statement below is false.

$$[(p \vee \neg r) \leftrightarrow (q \rightarrow r)] \vee r$$

Solution: The statement is false if and only if r is false, q is true, and p is either true or false.

2. Prove that the argument below is valid. (9 pts.)

$$\frac{\begin{array}{c} p \vee \ (q \rightarrow r) \\ \neg q \rightarrow \neg r \end{array}}{\neg p \rightarrow (q \leftrightarrow r)}$$

Solution: Suppose both premises are true. There are two cases to consider, the case where p is true and the case where p is false. If p is true, then the conclusion, which is an implication whose supposition is false, must be true. If p is false, then for the first premise to be true $q \rightarrow r$ must be true. But then since the second premise is equivalent to $r \rightarrow q$, we conclude that $q \leftrightarrow r$ is true in this case. Hence, the conclusion is true in this case as well.

□

3. True/false section. Circle one. No justification required. (2 pts. each)

True ***or False.*** For every real number x there exists a natural number n such that $n > x$.

True ***or False.*** There exists non-empty sets A, B such that $A \cap B \supseteq B$.

True or False. For every sets A, B, C we have $A \cap (B \cap C)^c = (A \cup B) \cap (A \cup C)$.

True or False. The statement $p \leftrightarrow q$ is false if p and q are both false.