Quiz 2 solution

1. Write down the converse of the implication below.

If f and g are one-to-one, then $g \circ f$ is one-to-one.

Define functions $f : \{1, 2\} \to \{a, b, c\}$ and $g : \{a, b, c\} \to \{1, 2\}$ which illustrate that the converse of the implication is false. (10 pts.)

Solution: The converse is *If $g \circ f$ is one-to-one, then both f, g are one-to one.* This statement is false as exhibited by the following example: take

$$f : \{1,2\} \to \{a,b,c\} \qquad g : \{a,b,c\} \to \{1,2\}$$
$$f(1) = a \qquad g(a) = 1$$
$$f(2) = b \qquad g(b) = 2$$
$$g(c) = 2$$

Then note that f is one-to-one but g is not. Also, $g \circ f$ is one-to-one since it is the identity map on A. Therefore, the converse of the implication is false since $g \circ f$ is one-to-one but it is not the case that both f, g are.

□

2. Prove that if A and B are subsets of a universal set U and $A \subseteq B$, then $B^c \subseteq A^c$. (10 pts.)

Solution: Suppose $A \subseteq B$ and we want to show that $B^c \subseteq A^c$. Let x be an arbitrary element of B^c and we show that x belongs to A^c.

Since $x \in B^c$, the element x does not belong to B. If x belonged to A, then since $A \subseteq B$ it must be that x belongs to B as well, which is impossible. Therefore, x must not belong to A. In other words, $x \in A^c$, as desired.

□