## Quiz 6

**1.** Find a recursive formula and a closed formula for the sequence below. (8 pts.)

3, 6, 9, 12, 15, . . .

Closed

$a_n = 3n, \; n \geq 1$

recursive

$i \geq 2 \quad a_n = a_{n-1} + 3, \; a_1 = 3.$

**2.** Prove that for every $n \geq 1$, (8 pts.)

$5^n - 3^n$ is even.

Proof: By induction.

Base case $n=1$ $\quad 5^1 - 3^1 = 2$ is even ✓

Inductive step Suppose $5^n - 3^n = 2k$ for some $k \in \mathbb{Z}$.

WTS $5^{n+1} - 3^{n+1} = 2l$ for some $l \in \mathbb{Z}$.

Set $l = 5^n + k$ ✓

Now, $5^{n+1} - 3^{n+1} = 5 \cdot 5^n - 3 \cdot 3^n = (2+3)5^n - 3 \cdot 3^n$

$= 2 \cdot 5^n + 3(5^n - 3^n) = 2 \cdot 5^n + 2k = 2(5^n + k).$

**3.** Formally state and give a proof of Junior's last statement in the conversation below. (4 pts.)

**Sal:** Do you know your multiples of 3?
**Junior:** Yes. They are 3, 6, 9, 12, 15, . . . .
**Junior:** If you add up the first forty of them you get 2460.

Statement

$3 + 6 + 9 + 12 + 15 + \ldots + 3(40) = 2460.$

Proof. $\text{LHS} = 3(1) + 3(2) + \ldots + 3(40)$

$= 3(1 + 2 + \ldots + 40)$

$= 3 \cdot \dfrac{40(40+1)}{2}$ $\quad$ (by Euler)

$= 60 * 41 = 2{,}460 = \text{RHS}.$ ✓